PATTERNS IN A COMPLEX SYSTEM: AN EMPIRICAL STUDY OF VALUATION IN BUSINESS BANKRUPTCY CASES

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This Article applies complex systems research methods to explore the characteristics of the bankruptcy legal system. It presents the results of an empirical study of twenty years of bankruptcy court valuation doctrine in business cramdown cases. The data provide solid descriptions of how courts exercise their discretion in valuing firms and assets.

This Article has two objectives: First, using scientific methodology, it explains the content of bankruptcy valuation doctrine. Second, the Article uses doctrine as a variable to explore the system dynamics that govern the processes of change over time.

Significant findings include: (1) Courts tend to split the difference in valuations much less frequently than expected; (2) while early data show debtors' and creditors' valuation positions were close together, later data show the parties' valuations moved further apart; (3) bankruptcy courts' valuation approach is substantially influenced by whether the valuation includes a calculation for the time value of money; (4) there seems to be some geographic distribution of courts' acceptance of valuation models, with courts in southern circuits more likely to accept soft valuation models, and nonsouthern circuit courts more likely to accept hard valuation models; and (5) the evidence offers preliminary support for the hypothesis that bankruptcy system content may self-organize according to some complex deterministic dynamics.

INTRODUCTION

Bankruptcy law is a complex, adaptive system.[1] Despite the presence of a complicated statute and a lengthy set of procedural rules, the bankruptcy system leaves much of its most important work up to the discretion of actors

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1. By "complex," I do not mean that the system has many moving parts, although bankruptcy certainly does. My sense of the term "complex" is borrowed from research in physics and other natural sciences. A system is "complex," in this sense, when it operates far from equilibrium. Equilibrium systems are characterized by balance, such that any flow of matter through the system represents a disturbance—a sort of crisis that must be managed so that the system may return to its characteristic balance. Far-from-equilibrium systems, on the other hand, are characterized by imbalance; these systems are paradigmatically poised on the edge of substantial change. Mathematically, the relationships between variables in a complex system tend to be nonlinear, the arithmetic is discrete (rather than continuous), and the geometry is fractal (rather than Euclidian). Most significantly for this project, the dynamics of complex systems are frequently observed to be self-organizing rather than entropic. A complex system endogenously adapts to environmental changes and tends to reach higher forms of order over time. Complex systems in nature include the ecological, such as a river bed, and the biological, such as a cell. The literature on complex systems is extensive, and these ideas are spreading from the study of natural systems to the study of social systems. See, e.g., PER BAK, HOW NATURE WORKS: THE SCIENCE OF SELF-ORGANIZED CRITICALITY 183–98 (1996); BENOIT B. MANDELBROT & RICHARD L. HUDSON, THE (MIS)BEHAVIOR OF MARKETS: A FRACTAL VIEW OF RISK, RUIN, AND REWARD (2004); JULIEN CLINTON SPROTT, CHAOS AND TIME-SERIES ANALYSIS (2003); Steven N. Durlauf, What Should Policymakers Know About Economic Complexity?, 21 WASH. Q. 157 (1998); Arthur F. McEvoy, Working Environments: An Ecological Approach to Industrial Health and Safety, 36 TECH. & CULTURE S145, S153–55 (1995); J.C. Sprott, Competition With Evolution in Ecology and Finance, 325 PHYSICS LETTERS A 329 (2004), available at http://sprott.physics.wisc.edu/pubs/paper285.pdf; M.E.J. Newman, Power Laws, Pareto Distributions and Zipf's Law (Jan. 9, 2005), http://arxiv.org/PS_cache/cond-mat/pdf/0412/0412004.pdf. Applications of complex systems science to legal phenomena are relatively recent and hold much promise. See Thomas A.C. Smith, The Web of Law (Spring 2005), available at http://ssrn.com/abstract=642863 (showing power law properties in the citation of legal materials as evidence that legal systems may be characterized by self-organizing criticality); see also Bernard Trujillo, Self-Organizing Legal Systems: Precedent and Variation in Bankruptcy, 2004 UTAH L. REV. 483, 528–43 (arguing that bankruptcy law is a self-organizing system).

"on the ground."[2] Observing how bankruptcy actors operate in a discretion-rich environment may offer important information about the structure and dynamics of legal systems generally.

Valuation in bankruptcy provides a useful vantage point from which we may observe this complex system in action. In a wide variety of circumstances, litigants argue for, and bankruptcy courts assign, monetary values for firms and assets. But the law of valuation in bankruptcy is radically underdetermined by hierarchical forces exogenous to the bankruptcy system, such as congressional and appellate court directives. As a result, bankruptcy actors are left substantially on their own.

One significant area of bankruptcy valuation is the business cramdown proceeding. In a typical business cramdown, a bankrupt corporation (the debtor) has filed a plan of reorganization to which at least one class of creditors has objected. The debtor requests that the bankruptcy court confirm the plan despite the objection, in effect "cramming the plan down" the throats of the objecting class of creditors.[3] The U.S. Bankruptcy Code (the Code) allows such a cramdown,[4] but only if a judge concludes that the plan gives the objecting class of creditors full value for its claims.[5]

For example, suppose that the debtor owes the creditor one million dollars. The debtor files a reorganization plan promising to pay the creditor 10 percent equity ownership in the reorganized firm. The creditor, preferring cash, objects to the plan. In the cramdown proceeding, the debtor argues that its plan gives full value for the creditor's claim because the value of the reorganized firm will exceed ten million dollars, and so the creditor gets 10 percent of that—or one million dollars plus interest—the full value of its claim. The creditor responds that the firm will be worth less than ten million dollars, and so

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2. *See* Trujillo, *supra* note 1, at 490–500, 509–12 (discussing institutional conditions fostering bankruptcy court discretion). For a nonexhaustive list of "on the ground" actors, see *id.* at 520–21.

3. *See generally* Kenneth N. Klee, *All You Ever Wanted to Know About Cram Down Under the New Bankruptcy Code*, 53 AM. BANKR. L.J. 133 (1979) (discussing the requirements for reorganization and the circumstances under which cramdown may occur).

4. 11 U.S.C. § 1129(b) (2000).

5. *See* 11 U.S.C. § 1129(b)(2). Full value means the allowed amount of the creditor's claim "as of the effective date of the plan," which amounts to a principal sum plus interest. The judge, in order to cram a plan down on a class of secured creditors (typically there is only one secured creditor per class), must find the plan to be "fair and equitable" under § 1129(b)(2)(A), which entails a finding that the plan distributes the full value of the secured creditor's claim. To cram a plan down on a class of unsecured creditors, the court must find the plan is "fair and equitable" under § 1129(b)(2)(B), which entails a finding that the plan distributes the full value to the dissenting class, or, in the alternative, distributes no value to classes junior to the dissenting class (this is known as the "absolute priority rule"). 11 U.S.C. § 1129(b)(2)(B)(ii) (2000 & Supp. 2005). Thus, for a court to cram a plan down on a dissenting unsecured class when that plan proposes a distribution to classes junior to the dissenting class, the judge must find that the plan distributes the full value to the dissenters.

the plan will not give the creditor full value for its claim. The bankruptcy court must, in effect, find a value under these circumstances for the reorganized firm.[6]

Projecting the value of a not-yet-reorganized firm is a complicated matter. Yet the Code and appellate court decisions provide very little guidance for this crucially important task.[7] Given the difficulty of the task and the lack of hierarchical ordering, one might expect valuation determinations either to (1) show no particular pattern, with decisions all over the map; or (2) follow a split-the-difference pattern, with judges typically selecting a number in between the debtor's number and the creditor's. The data, surprisingly, do not confirm these expectations.

I have concluded an empirical study of twenty years of bankruptcy court doctrine, covering all corporate cramdown cases decided from 1979, the first year after the enactment of the Code,[8] through 1998. While lawyers and legal scholars have offered theoretical and normative accounts of valuation in bankruptcy,[9] there remains a need for simple description. What do bankruptcy

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6.   This example is based on a cramdown under 11 U.S.C. § 1129(b)(2)(A) (2000).

7.   See, e.g., 11 U.S.C. § 506(a) ("Such value shall be determined in light of the purpose of the valuation and of the proposed disposition or use of such property . . . ."); Till v. SCS Credit Corp., 541 U.S. 465 (2004) (failing to provide clear standards for valuing interest rate); Assocs. Commercial Corp. v. Rash, 520 U.S. 953 (1997) (failing to provide clear standards for valuing principal).

8.   The enactment of the Bankruptcy Code in 1978 is an important exogenous event that reset the bankruptcy system. Such a system reset put us in a position to observe a transient approach to an attractor, as the system self-organized from the initial state. The enactment of the Bankruptcy Reform Act on April 20, 2005 was a similar system reset that once again will allow us to observe how the bankruptcy system self-organizes. See SPROTT, supra note 1, at 30–31 (discussing transient chaos).

9.   Works treating bankruptcy valuation include John B. Butler, III, Valuation of Secured Claims Under 11 U.S.C. 506(a), 89 COM. L.J. 342 (1984); David Gray Carlson, Secured Creditors and the Eely Character of Bankruptcy Valuations, 41 AM. U. L. REV. 63 (1991); John Collen, Real Estate Valuation Techniques, 8 J. BANKR. L. & PRAC. 135 (1999); Judith Elkin et al., Cramdown Interest: Formulas for Determining Risk, Rate and Return, 14 J. BANKR. L. & PRACT. 23 (2005); Chaim J. Fortgang & Thomas Moers Mayer, Valuation in Bankruptcy, 32 UCLA L. REV. 1061 (1985); Robert M. Lawless & Stephen P. Ferris, Economics and the Rhetoric of Valuation, 5 J. BANKR. L. & PRAC. 3 (1995); Frederick M. Luper & Kenneth M. Richards, Valuation of Property Issues in Bankruptcy, 98 COM. L.J. 35 (1993); Peter V. Pantaleo & Barry W. Ridings, Reorganization Value, 51 BUS. LAW. 419 (1996); James F. Queenan, Jr., Standards for Valuation of Security Interests in Chapter 11, 92 COM. L.J. 18 (1987); Kaaran E. Thomas, Valuation of Assets in Bankruptcy Proceedings: Emerging Issues, 51 MONT. L. REV. 126 (1990). See generally 7 COLLIER ON BANKRUPTCY ¶ 1129.06[2] (Alan N. Resnick & Henry J. Sommer eds., 15th ed. 2005) (recounting the history of current practices in Chapter 11 valuations). For an overview of valuation in various legal contexts, see Jay W. Eisenhofer & John L. Reed, Valuation Litigation, 22 DEL. J. CORP. L. 37 (1997). The finance literature has also treated the area of bankruptcy valuation. See generally David T. Brown, Claimholder Incentive Conflicts in Reorganization: The Role of Bankruptcy Law, 2 REV. FIN. STUD. 109 (1989); Pascal Francois & Erwan Morellec, Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures, 77 J. BUS. 387 (2004); Ronald M. Giammarino, The Resolution of Financial Distress, 2 REV. FIN. STUD. 25 (1989); Stuart C. Gilson et al., Valuation of Bankrupt Firms, 13 REV. FIN. STUD. 43 (2000); Robert E. Kalaba et al., Estimation of Implicit Bankruptcy Costs, 39 J. FIN. 629 (1984).

courts do when asked to value? We find that, despite the lack of exogenous ordering, stable and navigable patterns of valuation have emerged.

Part I of this Article discusses the study of doctrine, considers some criteria for studying doctrine empirically, and lays out the design of this study. Part II presents the statistical results of this study and some interpretations of those results. Part III summarizes the major findings and concludes with some questions and directions for future research.

I. STUDYING DOCTRINE EMPIRICALLY

There are at least three reasons for studying doctrine:[10] (1) We want to know what the doctrine is; (2) we want to learn about the behavior of persons or institutions, and we think doctrine offers some explanation of that behavior; and (3) we want to explain what a legal system is and how it works, and we use doctrine as a variable to test propositions regarding a legal system's structure and dynamics.

In the first type of study (what is the doctrine?), researchers regard doctrine as the dependent variable, or the thing-to-be-explained. Researchers consider independent variables in the hope of explaining, and perhaps predicting or ultimately reforming, legal doctrine. These independent variables can be both formal (for example, jurisdiction, time, judge, type of case, and party structure) and realist (for example, politics, economic incentives, race, class, gender, and social norms[11]).

In the second type of research (what is the behavior of persons and institutions?), the thing-to-be-explained is behavior. Doctrine serves as one among many independent variables that may shed light on behavioral hypotheses.

In the third type of doctrinal study (what are the dynamics of a legal system?), researchers again treat doctrine as an independent variable that may help explain a targeted thing-to-be-explained. That thing, however, is not some behavioral pattern, but rather the structure and dynamics of the legal system itself.[12]

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10. Doctrine is the body of formally defined rules of a legal system. These rules are officially promulgated by the authorized agents of the system, and include statutes, court opinions, and administrative regulations. A common feature of doctrine is that it is written, published, and available to the public. As a simplification, I will be taking the published judicial opinion as the central example of doctrine for the purposes of this Article.

11. *See* Marc Galanter, *The Portable Soc 2; or, What to Do Until the Doctrine Comes, in* GENERAL EDUCATION IN THE SOCIAL SCIENCES: CENTENNIAL REFLECTIONS ON THE COLLEGE OF THE UNIVERSITY OF CHICAGO 246, 252 (John J. MacAloon ed., 1992) ("I make no distinction between believers in the model of rules and instrumentalists; nor between formalist believers in autonomous rule development and their realist critics. Thus, where some observers detect a radical break, I see a striking continuity.").

12. *See infra* Part II.E.3 (discussing randomness versus chaos in system dynamics).

This project intends no contribution to the second type of doctrinal research. The data in this study—published bankruptcy court cases—cannot credibly falsify any behavioral hypotheses. We cannot, on the basis of this work, draw conclusions about what backroom deals are struck or what disputes are not brought to formal adjudication. This project does, however, hope to contribute to both the first and third types of doctrinal study.

A. Explaining Doctrine

One objective of this research is to understand more about the doctrine of bankruptcy valuation. Two tendencies mark my pursuit of this objective: First, this Article suspends discussion of any normative theory of value[13] in favor of straight description. Much doctrinal research is at least as interested in what the doctrine *ought* to be as in what the doctrine *is*.[14] By contrast, this Article will try to refrain from imposing, ex ante, any overarching theory upon the data.[15]

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13.   For useful general theorizations of valuation, see, for example, Margaret Jane Radin, *Compensation and Commensurability*, 43 DUKE L.J. 56 (1993); Cass R. Sunstein, *Incommensurability and Valuation in Law*, 92 MICH. L. REV. 779 (1994); Cass R. Sunstein et al., *Assessing Punitive Damages (With Notes on Cognition and Valuation in Law)*, 107 YALE L.J. 2071 (1998). For two of the more useful articles theorizing valuation in the bankruptcy area, see Carlson, *supra* note 9, and Lawless & Ferris, *supra* note 9.

14.   This blurring of *is* with *ought* is characteristic of what John Griffiths calls an "ideological" rather than an "empirical" position. *See* John Griffiths, *What Is Legal Pluralism?*, 24 J. LEGAL PLURALISM 1, 3 (1986). "Ideology," according to Griffiths, is a "mixture of assertions about how the world ought to be and *a priori* assumptions about how it actually and even necessarily is." *Id.* "Empirical," descriptive, or scientific approaches to the study of law, on the other hand, take the phenomenon as the primary object of study, without entertaining questions about how the phenomenon ought to be or what it necessarily must be. *Id.*; *see also* Galanter, *supra* note 11, at 251–53 (listing eight propositions of conventional legal studies. Galanter also noted that the listed propositions

> have a dual, composite character, fusing both descriptive and normative. They are thought
> to state what is normal and typical in legal systems—to reflect the inherent and proper shape
> of legal reality. This fusion of factual and normative assertion . . . establishes them as ideo-
> logical statements—statements about what a legal system . . . ought to be like.

Id.

15.   *See* Comm'r v. Marshall, 125 F.2d 943, 946 (2d Cir. 1942). In the opinion of the court, Judge Jerome Frank wrote:

> The fallacy . . . stems largely from lack of recognition of the eely character of the word
> "value." It is a bewitching word which, for years, has disturbed mental peace and caused
> numerous useless debates. Perhaps it would be better for the peace of men's minds if the
> word were abolished. Reams of good paper and volumes of good ink have been wasted by
> those who have tried to give it a constant and precise meaning.

Id.; *see also* Old Colony Bondholders v. N.Y., N.H. & H.R. Co., 161 F.2d 413, 450 (2d Cir. 1947) (Frank, J., dissenting) ("[I]t would be desirable to abandon the word 'valuation'—since that word misleadingly connotes some moderately rational judgment—and to substitute some neutral term, devoid of misleading associations, such as 'aluation,' or, perhaps better still, 'woosh-woosh.'"). Heeding the pleas of Judge Frank, this Article attempts description rather than theorization.

This Article revolves around the data, and the data show only what bank-ruptcy courts have memorialized about their valuation decisions.[16]

Second, this Article attempts to apply scientific methodology to doc-trinal study by treating doctrine as a quantitative unit. This methodological approach works particularly well in studying bankruptcy, which is an almost ideal laboratory for the empirical study of legal systems. In part because of the existence of over 330 specialty bankruptcy courts, the U.S. bankruptcy system emits a tremendous amount of visible data relative to many other legal systems. A system that emits large amounts of easily detectable data can be studied scientifically and quantitatively, without resort to anecdote or idiosyncrasy, and without the biases of expectation or experience.[17] This Article thus tries to make transparent all choices regarding selection and interpretation of data,[18] and uses statistics to interpret the sample data drawn from this complex system-in-motion.[19]

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16. Nothing in this study tells us how bankruptcy courts perform valuations in contexts beyond Chapter 11 cramdowns. We cannot, for example, generalize from these data to draw con-clusions about areas of consumer bankruptcy valuation. These data only show the variation of forms that exist in the doctrine of business cramdown valuations.

17. The methodological approach of this research follows the inspiration of the evidence-based medicine (EBM) approach to the practice of medicine. EBM promotes the scientific study of the efficacy of medical treatments. Such research allows medical professionals to prescribe treatment, not because of tradition or path dependence, but because of a well-informed belief that the treatment will be effective. *See generally* DAVID L. SACKETT ET AL., EVIDENCE-BASED MEDICINE: HOW TO PRACTICE AND TEACH EBM (1997). A frequently updated database of evidence is available for practitioners of EBM at http://www.clinicalevidence.com/ceweb/conditions/index.jsp. I believe that an evidence-based law approach to doctrine can move us past anecdote and unexamined path dependence, and perhaps toward a systematization and verification of knowledge about legal doctrine.

18. Often, doctrinal research bases its conclusions on a set of data that is both highly selective and rather small relative to the total amount of available data—for example, by limiting the data set to a few appellate court opinions—and typically does not explain its criteria for data selection and interpretation. By scientific standards, both the selection and review of such data are idiosyncratic, and the idiosyncrasies are not expressly revealed. As a consequence, such research cannot be rep-licated by other scholars.

19. There is an interesting similarity between the rise of statistical methods in the physical sciences and the roughly contemporaneous rise of impressionist methods in the visual arts. Both are ways of representing systems-in-motion, and of knowing those systems by sampling them. Henri Matisse wrote an essay entitled *Exactitude Is Not Truth* to accompany four sketches of his own face in a mirror. Matisse makes the point that the four renditions are all very different, but unmistakably represent the same subject. They are, if you will, four samples of the same complex system. *See* Henri Matisse, *Exactitude Is Not Truth*, in THEORIES OF MODERN ART 137–39 (Herschel B. Chipp ed., 1968). Matisse explained:

These drawings seem to me to sum up observations that I have been making for many years on the characteristics of a drawing, characteristics that do not depend on the exact copying of natural forms, nor on the patient assembling of exact details, but on the profound feeling of the artist before the objects which he has chosen . . . .

. . . [T]he leaves of a tree—of a fig tree particularly—the great difference of form that exists among them does not keep them from being united by a common quality. Fig leaves, whatever fantastic shapes they assume, are always unmistakably fig leaves. . . .

B.   Using Doctrine to Explain Legal Systems

The second objective of this research is to use doctrine to explain the dynamics of the bankruptcy legal system. At the threshold, we must establish whether doctrine is actually probative as evidence of a legal system's dynamics. Doctrine, understood here as the published opinions of bankruptcy courts, offers only uncertain support for some types of propositions. If, for example, we want to describe features of the entire population of bankruptcy disputes, then study-ing doctrine gives little or no help.[20]

Suppose, on the other hand, that we are interested in describing how the bankruptcy system works—how networks composed of various actors, such as judges, litigators, clients, and potential clients,[21] receive and transmit forms,[22] and how some forms acquire normative status while others do not. If we want to describe the dynamics of a legal system, then published opinions are singularly probative as data, because published opinions are an important "communications device"[23] that travel among the elements of the system, like

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Thus there is an inherent truth which must be disengaged from the outward appear-
ance of the object to be represented. This is the only truth that matters.
. . . .
    L'exactitude n'est pas la vérité.
Id. I thank Bill Clune for bringing this remarkable source to my attention.
    20.   *See, e.g.,* Peter Siegelman & John J. Donohue III, *Studying the Iceberg From Its Tip: A Comparison of Published and Unpublished Employment Discrimination Cases,* 24 LAW & SOC'Y REV. 1133 (1990).
    21.   *See* Trujillo, *supra* note 1, at 520–21 (presenting a players list for the U.S. bankruptcy legal system).
    22.   "All forms are simply data that make up [a given legal] system." *See, e.g.,* Trujillo, *supra* note 1, at 486 n.1 (defining "forms" for the purposes of this research, as ranging from the formal and institutional, to the informal and ephemeral).
    23.   Martin Shapiro's work is fundamental in establishing the communications aspect of judi-cial opinions. Shapiro observes:
        In recent years political science has focused not on judicial opinions but on judicial deci-
        sions (who won and lost) as keys to understanding judicial attitudes. In legal theory . . . there
        has been much attention to judicial opinions as justifications or explanations . . . . But
        nearly all the commentators . . . treat the opinion *in vacuo,* asking whether it meets certain
        general standards and thus turning the problem into one of logic or philosophy.
        Somehow we ignored the fact that appellate courts and the lawyers that serve them
        spend an overwhelming proportion of their energies in communicating with one another,
        and that the judicial opinion . . . is the principal mode of communication . . . .
        It would seem appropriate, therefore, to examine the opinion-writing activity of courts
        in the context of communication . . . .
Martin Shapiro, *Toward a Theory of Stare Decisis,* 1 J. LEGAL STUD. 125, 134 (1972); *see also* MALCOLM M. FEELEY & EDWARD L. RUBIN, JUDICIAL POLICY MAKING AND THE MODERN STATE: HOW THE COURTS REFORMED AMERICA'S PRISONS 211–52 (1998). Feeley and Rubin write:
        Probably the most common, and certainly the most studied, means of [judges communi-
        cating ideas to one another] is a judicial opinion . . . . In writing the opinion, the judge sets
        down her public account of the way that her personal beliefs can be integrated with her

proteins in a cell.[24] Judges intend their published opinions not only as a communication to the parties in the particular case that gave rise to the opinion, but also as a communication to other judges,[25] other lawyers, other litigants, and other actual and potential participants in the legal system.[26]

The selection process that precedes publication makes doctrine even more probative of system dynamics. Judges determine when to write an opinion down instead of delivering it orally from the bench. Judges also determine which, from among their written opinions, they will send to the publisher.[27] Published opinions thus represent a subset of all written opinions, which in turn represent a subset of all formal adjudications, which in turn represent a subset of all dispute resolutions. Behavioral scholars point to this winnowing process to bolster their claim that published opinions are freighted with judicial bias and are thus very poor evidence of what is really happening. This claim is correct if the target of research is the entire pool of disputes to be resolved. But if the purpose of the research is to study internetwork and intranetwork communications signals, then this winnowing process improves the sample rather than biases it. Bankruptcy valuation doctrine is useful evidence for studying system dynamics

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perception of existing legal doctrine. That is a thought process of its own, born of the need to act consistently with one's role expectations. It represents the judge's real feelings in her institutional role, and the question of whether it represents her real feelings about the integrative effort, apart from the personal attitudes that have already been incorporated in the integration process, is too metaphysical to be productive.

Edward Rubin & Malcolm Feeley, *Creating Legal Doctrine*, 69 S. CAL. L. REV. 1989, 2012 (1996); *see also* MARC GALANTER, LAW AND SOCIETY IN MODERN INDIA 3–11 (1989) (on using doctrine in the field of Indian legal studies). *See generally* CLAUDE E. SHANNON & WARREN WEAVER, THE MATHEMATICAL THEORY OF COMMUNICATION (1963) (a seminal work exploring the mathematical structure of communications systems).

24. Other communications devices in legal systems include contracts, *see* Stewart Macaulay, *Non-Contractual Relations in Business: A Preliminary Study*, 28 AM. SOC. REV. 55, 65 (1963) (describing how "a fairly detailed contract can serve as a communication device within a large corporation"), speeches, and articles, *see* Rubin & Feeley, *supra* note 23, at 2012–13 ("[O]ther ways for judges to communicate . . . includ[e] attending judicial conferences, making personal contacts, serving on multimember panels, and stating their views in law reviews and similar publications. In a concrete, anti-formalist account of judging, such means of communication count just as much as the traditional means of written opinions.").

25. Rubin & Feeley, *supra* note 23, at 2016 ("Judges do not meet together very often . . . . For the most part, they communicate [with each other] through their written opinions . . . .").

26. *See, e.g.*, Andrew P. Morriss et al., *Signaling and Precedent in Federal District Court Opinions*, 13 SUP. CT. ECON. REV. 63 (2005) (discussing judges' use of opinions to signal to appointing authorities).

27. Criteria for publishing opinions vary between individual judges. A judge may publish opinions on issues that are particularly important, difficult, or novel. A judge may publish opinions on pedestrian, oft-repeated claims as a way of signaling her tendencies to the bar and encouraging settlement. A judge may publish, and write with extreme care, opinions in cases she feels will be appealed, perhaps because of the parties' makeup. All of these reasons improve the sample in that they represent the judges' ex officio speech to a particular audience—future litigators in her own district, fellow judges in other districts, and appellate judges.

precisely because it is not an unvarnished report of an objective event—what was the found value of the asset?—but rather a subjective account of an objec-tive event—what did the judge say about finding the value of the asset?[28]

If doctrinal data can explain something about system dynamics, then the data of this study may illuminate at least two features of the bankruptcy legal system: (1) the effects of discretion; and (2) the tendency to self-organization.

Bankruptcy law contains a high amount of judicial discretion.[29] Scholars and policymakers often paint judicial discretion as a problem, in part because the outcomes of a discretion-rich system are, at least in theory, uncontrolled and subject to caprice. Yet despite the theoretical possibility that anything can happen in a discretion-rich space, the data in this Article show that, in fact, just anything does not happen. Rather, very constrained and navigable pat-terns have emerged in the area of bankruptcy valuation.[30] This Article hopes to make transparent how bankruptcy courts use their discretion in the area of valuation. As a more general aspiration, one may hope that by measuring the uses and effects of judicial discretion in a specific area, legal actors and com-mentators may begin to understand and become comfortable with discretion as an omnipresent and unremarkable feature of complex legal systems.[31]

Another characteristic of bankruptcy system dynamics is self-organization, or the emergence of patterns that are not attributable to exogenous, extrasys-temic events. The data in this Article, echoing other empirical bankruptcy work,[32] show that the bankruptcy legal system produces stable and navigable patterns of behavior, despite the absence of rigorous hierarchical control by appellate courts and Congress.

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28. Studying doctrine to understand system dynamics is a content-neutral use of doctrine, in the sense that we will learn about the system's dynamics regardless of the doctrine's content—regardless of whether a given court, say, adopted or rejected a particular discounted cash flow model. Such research is designed not necessarily to predict future system content, but rather to describe the processes that produce future system content. See, e.g., RAYMOND T. NIMMER, THE NATURE OF SYSTEM CHANGE: REFORM IMPACT IN THE CRIMINAL COURTS 3 (1978) ("My premise is that the reform process is an inde-pendently significant phenomenon, typified by common patterns and delimiting characteristics. This phenomenon can and should be studied quite apart from the specific goals of particular reforms.").

29. See Trujillo, supra note 1, at 490–500, 509–12.

30. See infra Part II.D.

31. See Edward L. Rubin, Discretion and Its Discontents, 72 CHI.-KENT L. REV. 1299, 1299–300 (1997) (suggesting that judicial discretion is presented as a problem, although it is actually a ubiquitous and unremarkable feature of modern law); Rubin & Feeley, supra note 23, at 2037 ("[J]udicial creation of law] can be described, understood, and justified. It is one of the basic, quotidian elements of our legal system.").

32. See, e.g., Jean Braucher, Lawyers and Consumer Bankruptcy: One Code, Many Cultures, 67 AM. BANKR. L.J. 501, 532 (1993); Lynn M. LoPucki, The Demographics of Bankruptcy Practice, 63 AM. BANKR. L.J. 289 (1989); Lynn M. LoPucki, Legal Culture, Legal Strategy, and the Law in Lawyers' Heads, 90 NW. U. L. REV. 1498, 1506–07 (1996); Teresa A. Sullivan et al., The Persistence of Local Legal Culture: Twenty Years of Evidence From the Federal Bankruptcy Courts, 17 HARV. J.L. & PUB. POL'Y 801, 804 (1994).

C. Design of This Study

The data for this Article are drawn from reported opinions of U.S. Bankruptcy Courts in Chapter 11 cramdown proceedings that contain a judicial valuation of at least one asset.[33] The database consists of 180 observations drawn from 145 published opinions reported in the Westlaw computer database, and decided from 1979 through 1998.[34] A law student, working closely with me, coded each observation according to a substantial coding instrument.[35] Following the initial coding, a substantial portion of the observations was recoded by a different law student to test for reliability.[36]

Besides the fact that the data are drawn from published cases,[37] other potential limitations of the database include the possibility that the computer search failed to identify relevant cases, that Westlaw misclassified some relevant cases, and that the coding process registered some false negatives (by failing to classify an item as an observation despite the presence of a reported judicial valuation). Thus, while these data support statistical conclusions about the population of all reported cases from 1979 through 1998, the present database is properly understood as only a sample of that population.

Each observation was coded for several variables. The meanings of most of the variables (such as date of adjudication or federal circuit) are obvious, but five variables merit some initial explanation: (1) party success; (2) party proximity; (3) finance element; (4) valuation standard; and (5) valuation model.

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33.   In this cramdown database, judicial valuations typically occurred in one of two contexts. In some, a dissenting class of creditors (typically a single secured creditor) sought cramdown rights under § 1129(b)(2)(A) through a judicial valuation of the collateral securing its claim. In others, the debtor's plan of reorganization proposed to extinguish a creditor's lien in a piece of collateral in exchange for compensation, and the court valued the item to determine if the compensation was appropriate and the plan was confirmable. The parties to these valuation proceedings were corporations.

34.   The Westlaw search term used to acquire the cases was "51K3563 51K3564 51K3565 & DA(AFT 1978 & BEF 1999)" in the library "fbkr-bct." This term yielded a list of 388 cases, which generated 180 observations, reported in 145 cases. Observations consist of a reported numerical valuation of an asset by a court. If a single case reported valuations of multiple assets, each asset was coded as a separate observation. The earliest observation is from December 5, 1980. The latest observation is from November 20, 1998.

35.   *See infra* Appendix (describing how a reader can access a copy of the coding instrument). A coding instrument is a questionnaire that a coder uses to transform written information into quantitative units. *See generally* HERBERT JACOB, USING PUBLISHED DATA: ERRORS AND REMEDIES (Sage University Papers Series: Quantitative Applications in the Social Sciences No. 07-042, 1984) (discussing methodology generally applicable to this study); ROBERT PHILIP WEBER, BASIC CONTENT ANALYSIS (Sage University Papers Series: Quantitative Applications in the Social Sciences No. 07-049, 2d ed. 1990) (same).

36.   *See infra* Appendix, tbl.1-A (presenting reliability statistics).

37.   *See supra* Part I.B (discussing the relationship between data's publication and its probative value).

(1)   Party success is a numerical representation of the degree to which the parties in the valuation proceeding won or lost.[38]

(2)   Party proximity is a numerical measure of how far apart the debtor and the creditor were in the dollar valuations they sought from the court.[39]

(3)   Finance element is a variable for whether the valuation contained a calculation or discussion of the time value of money. Of the 180 observations, 74 (41 percent), contained a finance element, and 106 (59 percent) did not.[40]

(4)   Valuation standard is a variable that attempts to represent the internal criteria used by a particular valuer (court, debtor, or creditor) when it assigned value to an item—for example, whether the valuer based the valuation on the item's anticipated use, or resale, or some mix between use and resale.[41]

(5)   Valuation model tracks the way that parties argue for a particular valuation, for example, by stating what an asset would be worth if it were liquidated (a liquidation model), or by constructing a prediction of what cash flows an asset would earn over time (a discounted cash flow model).[42]

II. STUDY RESULTS AND INTERPRETATIONS

This part of the Article presents the conclusions of statistical analysis of the data, and offers some interpretations of the results.

A. Party Success

The database illuminates some patterns of party success in valuation proceedings. We define success quantitatively, by comparing the valuation proposed by the parties with the valuation found by the court. Using two

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38.   *See infra* Part II.A for the mathematical construction of the success variable and analysis.
39.   *See infra* Part II.B for the mathematical construction of the party proximity variable and analysis.
40.   A case was coded as containing a finance element if the court, as part of the valuation, also determined an interest rate or discount rate.
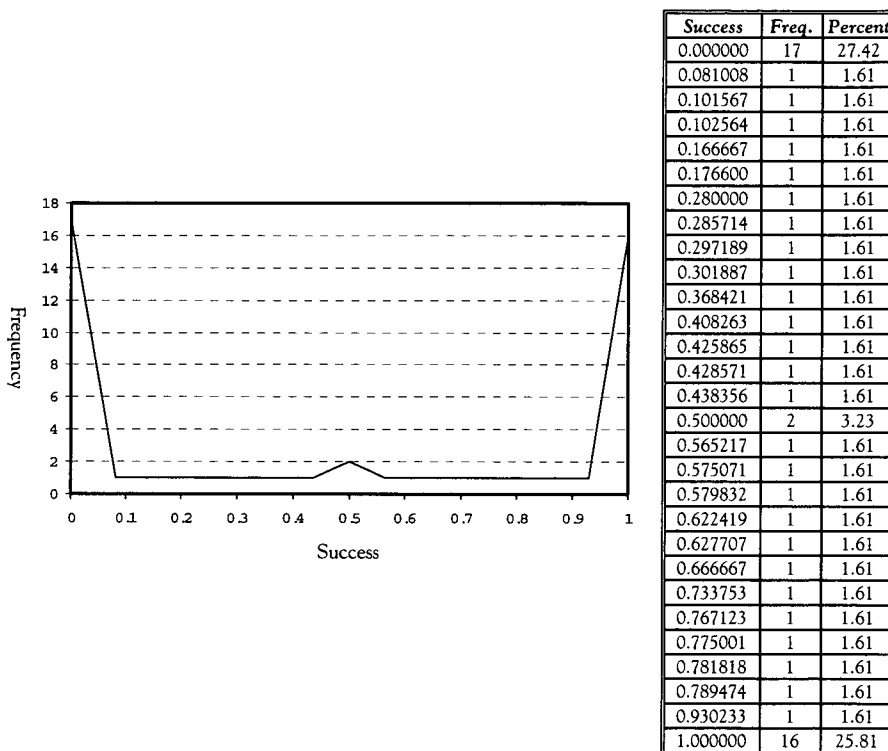41.   *See infra* Part II.B for the mathematical construction of the valuation standards variable and analysis.
42.   *See infra* Part II.C for further analysis of valuation models.

simple mathematical formulae,[43] we describe the range of party success from "1" (debtor received all that it sought) to "0" (creditor received all that it sought).

Figure 1 shows the frequency of values for the success variable.

FIGURE 1

PARTY SUCCESS: FREQUENCY DISTRIBUTION

Success	Freq.	Percent
0.000000	17	27.42
0.081008	1	1.61
0.101567	1	1.61
0.102564	1	1.61
0.166667	1	1.61
0.176600	1	1.61
0.280000	1	1.61
0.285714	1	1.61
0.297189	1	1.61
0.301887	1	1.61
0.368421	1	1.61
0.408263	1	1.61
0.425865	1	1.61
0.428571	1	1.61
0.438356	1	1.61
0.500000	2	3.23
0.565217	1	1.61
0.575071	1	1.61
0.579832	1	1.61
0.622419	1	1.61
0.627707	1	1.61
0.666667	1	1.61
0.733753	1	1.61
0.767123	1	1.61
0.775001	1	1.61
0.781818	1	1.61
0.789474	1	1.61
0.930233	1	1.61
1.000000	16	25.81

43. To calculate the success variable (denoted S in the following equations), we took observations that contained numerical expressions of (1) the debtor's requested value (D); (2) the creditor's requested value (C); and (3) the court's found value (Ct). Values of D and C must be unique; $n = 62$. Where $C > D$, $S = C–Ct / C–D$. To illustrate, suppose $C = 10, D = 6, Ct = 9$. So $10–9 / 10–6 = 0.25$ (a number closer to 0 than 1, indicating C prevailed). On the other hand, suppose $C = 10, D = 6, Ct = 7$. So $10–7 / 10–6 = 0.75$ (indicating D prevailed). Where $C < D$, $S = Ct–C / D–C$. To illustrate, suppose $C = 6, D = 10, Ct = 9$. So $9–6 / 10–6 = 0.75$ (D prevails). On the other hand, suppose $C = 6, D = 10, Ct = 7$. So $7–6 / 10–6 = 0.25$ (C prevails). Thanks to Menesh Patel for help in developing this version of the success variable.

This figure shows a snapshot aggregation of success patterns, without breaking down the variable by time.[44] The curve in Figure 1 looks a bit like a "W," describing a pattern in which the court picks either the debtor's number or the creditor's number—thus, complete success for the debtor[45] or for the creditor[46]—about equally. Remarkably, courts very rarely split the difference between the debtor's and the creditor's numbers, a position represented by the small hump in the middle of the W-curve.

These findings are somewhat surprising, in that they upset the common wisdom that courts frequently resolve valuation disputes by splitting the difference between the offered numbers.[47] If the common wisdom were true, then the curve in Figure 1 would resemble the letter "A" (high observations in the middle with lower observations on either end)[48] rather than the letter "W" (high observations on either end with only a little bump in the middle). The W-curve of Figure 1 thus is unexpected.[49] It indicates that judges engage in simple baby-splitting much less frequently than had been believed.[50]

Figure 2 shows a time series of those observations where the court followed a winner-take-all strategy by awarding total success to one party or the other. Figure 2 is thus a time series representation of the two extreme poles from the W-curve of Figure 1.

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44. For ease of presentation, one value of 0.003546 was omitted.

45. Represented in the chart by the sixteen "1s."

46. Represented in the chart by the seventeen "0s."

47. *See, e.g.,* Louis W. Levit, *Secured Claims, in* 2 CHAPTER 11 THEORY AND PRACTICE: A GUIDE TO REORGANIZATION 15:1, 15:12 (James F. Queenan, Jr. et al. eds., 1994). Levit explains valuation proceedings as follows:

> Unless the parties can agree [on a valuation], they must resort to the modern version of "trial by battle" in which the contenders are not gladiators or knights in armor but expert (and expensive) appraisers. Each appraiser will trot out his credentials and carefully prepared report showing the usual charts and illustrations designed to bolster the conclusions which will give his side what it wants. When the trial is concluded, the court will probably "cut the baby in half," and end up somewhere in the middle.
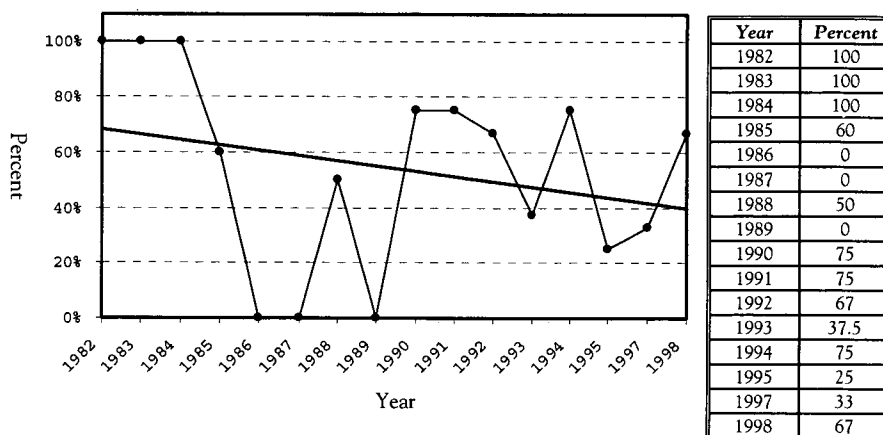
Id.

48. It could also be a head and shoulders curve, with slightly higher observations at the poles and much higher observations in the middle.

49. It is important to note that, since the database consists of published bankruptcy court cases, the data do not exclude the interpretation that courts officially signal (with their published opinions) a posture of "no baby-splitting" while continuing to baby-split in their unpublished opinions. Thanks to Bill Whitford for making this point clear to me. Such a result, if true, might make an interesting point about *systematic* differences between published and unpublished opinions. Evidence of any systematicity in the outcomes of published versus unpublished opinions would be a significant contribution to the elucidation of trial court dynamics.

50. Figure 1 resembles the decision pattern that would have been predicted under the (short-lived) rule announced by Chancellor Allen in *Cede,* in which he proclaimed that henceforth he would accept only one party's valuation model. Cede & Co. v. Technicolor, Inc., No. Civ. A. 7129, 1990 WL 161084, at *32 (Del. Ch. Oct. 19, 1990), *rev'd,* 684 A.2d 289, 300 (Del. 1996). *See infra* note 54 (discussing the problem of litigant overclaiming in Shapiro's work).

FIGURE 2
WINNER-TAKE-ALL ADJUDICATIONS OVER TIME

Year	Percent
1982	100
1983	100
1984	100
1985	60
1986	0
1987	0
1988	50
1989	0
1990	75
1991	75
1992	67
1993	37.5
1994	75
1995	25
1997	33
1998	67

While these data are not as well behaved as the data in the W-curve, one can detect a slight downward trend over time. Thus, while courts continue to deploy the winner-take-all strategy frequently, there has been a slight trend toward splitting the difference in the litigants' numbers.
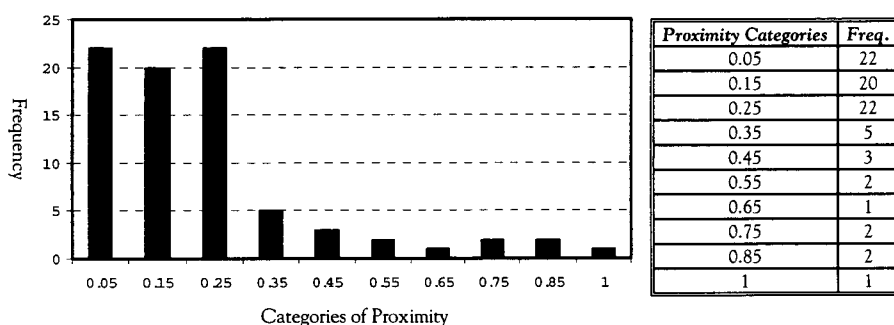
B. Party Proximity

The database also illuminates parties' valuation strategy. We construct a party proximity variable to measure the distance between dollar positions staked out by the parties in the valuation proceeding.[51] Values close to "0" indicate that the parties' dollar positions were very close together. Values approaching "1" indicate that the parties' dollar positions were very far apart. The party proximity variable is useful because it permits a dollars-to-dollars comparison without that comparison being distorted by differences in the time, place, or circumstances of the valuation. In this way, we can focus on the relative valuation strategies of the parties.

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51. Observations must have dollar positions for both the debtor and the creditor, and the positions must be comparable. Thus, we omit from the database valuation positions such as "3% over prime v. 11% over prime" because there is no constant way to compare these values. Where a party offered a range for its dollar position, we took the midpoint of that range; $n = 80$. The formula for party proximity is *(debtor's dollar position − creditor's dollar position) / (debtor's dollar position + creditor's dollar position)*. For example, in an observation from 1989, the debtor argued for a valuation of $9.2 million, and the creditor asked for $14.5 million. Applying the formula yields a quantity of −0.223629. For purposes of determining proximity, we took the absolute value of each quantity. I thank David Merrill for help in developing this version of the party proximity variable.

Figure 3 shows the frequency distribution of the party proximity variable.

FIGURE 3
PARTY PROXIMITY: FREQUENCY

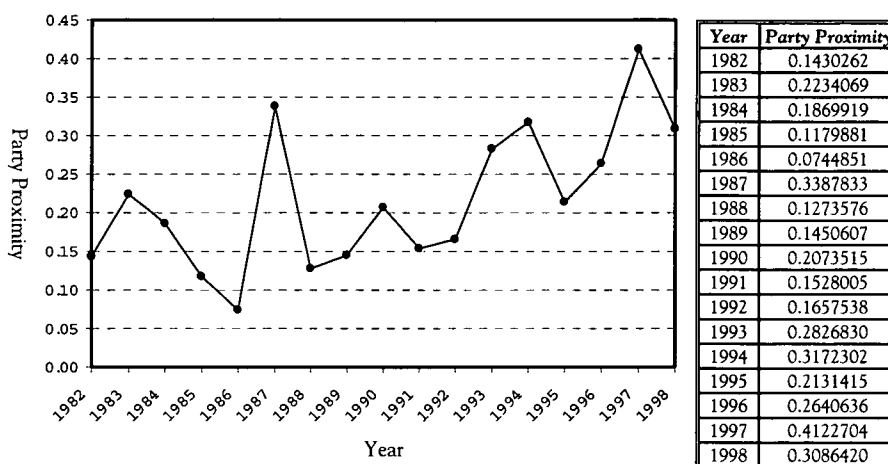| Proximity Categories	Freq.
0.05	22
0.15	20
0.25	22
0.35	5
0.45	3
0.55	2
0.65	1
0.75	2
0.85	2
1	1

Categories of Proximity

For example, there were twenty-two observations in which the party proximity was 0.05 or below (very close together). There was one instance in which the party proximity was above 0.85;[52] for the most part, parties' dollar positions tended to be close together.

Considering the data over time, however, we see a clear trend away from litigants' arguing for numbers that are close together. Figure 4 shows the mean values per year for party proximity.

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52.   This largest observation, from 1993, was 0.8930. In that case, the debtor sought a valuation of $23 million and the creditor argued for $1.3 million.
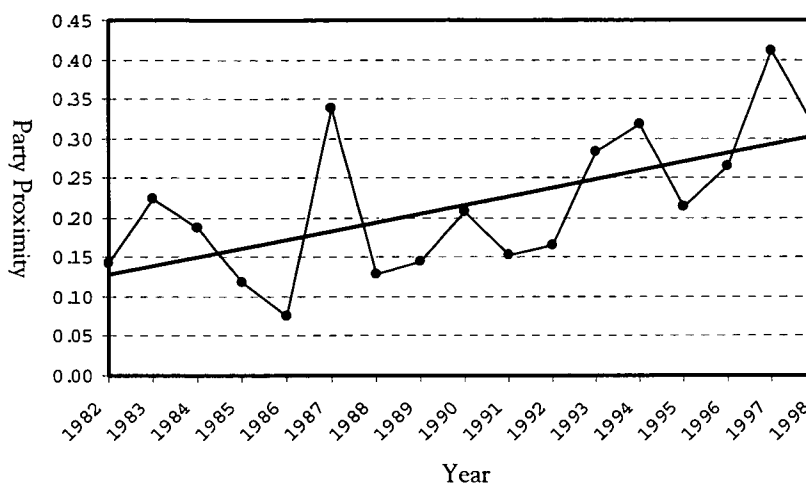
FIGURE 4
PARTY PROXIMITY OVER TIME: DESCRIPTIVE

Year	Party Proximity
1982	0.1430262
1983	0.2234069
1984	0.1869919
1985	0.1179881
1986	0.0744851
1987	0.3387833
1988	0.1273576
1989	0.1450607
1990	0.2073515
1991	0.1528005
1992	0.1657538
1993	0.2826830
1994	0.3172302
1995	0.2131415
1996	0.2640636
1997	0.4122704
1998	0.3086420

As a matter of descriptive statistics, it appears that parties' valuation positions moved farther apart over time. A standard linear regression, shown in Figure 5, demonstrates that the observed increase over time in the party proximity variable is statistically significant.[53]

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53.   The slope estimate was 0.0100108 with a standard error of 0.0018757 and a probability of 0.000 (t-value of 5.34). This means that for each additional year, the value of the party proximity variable increased by approximately 0.01. A t-test is a simple statistical test that allows us to state the likelihood that an observed correlation between two variables could have been observed merely by chance.

FIGURE 5

PARTY PROXIMITY OVER TIME: INFERENTIAL

Interpreting Figures 1–5, we can say that courts generally have followed a winner-take-all strategy, with perhaps some slight drift away from that strategy over time. Parties' dollar positions, generally close together early in the database, gradually drifted further apart over time, and that trend is statistically significant. Taking the courts' and the parties' valuation tendencies together, we see that early in the database, courts followed a winner-take-all strategy and parties' dollar positions were close together. Later in the database we may see some trend towards courts utilizing a split-the-difference strategy, while we see a significant trend towards parties' moving their dollar positions farther apart, following a highball/lowball strategy.[54] Courts' awarding of success and parties' litigation positions are thus mutually reinforcing: When courts pick a number, parties position their numbers close together; when courts split the difference, parties maximize the difference by placing their numbers far apart.[55] We cannot, of course, tell from the data whether parties are following the courts' lead or forcing the courts' hand.

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54. A strategy of highball/lowball occurs when the party seeking a low number presents a valuation model arguing for the lowest number that can possibly be pronounced with a straight face, and the party seeking a high number presents valuation evidence requesting the highest number that can be requested without incurring a Rule 11 violation. In this way, the parties' numbers are pushed to the extremes, and the usefulness of their valuation arguments for the court thus is diminished. *See Cede & Co.*, 1990 WL 161084, at *7–*9 & n.17 (Chancellor Allen attempting to defuse parties' highball/lowball strategy).

55. This finding supports the hypothesis, advanced by Martin Shapiro, that mediation (which promotes split-the-difference outcomes), tends to drive parties' claims further apart. *See* MARTIN

C. Valuation Standards

1. Definitions and Descriptive Statistics

The valuation standard is a variable constructed to represent the criteria used by a given valuer (court, debtor, or creditor) when it assigned value to an item. The variable is constituted across three categories, reflecting whether the valuer based the valuation on the item's anticipated use (independent value), resale (common value), or some mix between use and resale (mixed value). These three categories attempt to represent the range of possible valuation approaches. Taken together, the three categories of this dependent variable express a theory of value—an account of what a valuer is doing when it assigns value to an item.

We take our polar definitions of common and independent value from the auction literature of economics:[56] A valuer's valuation is independent when it is not influenced by other valuers' valuation of the same item.[57] For example, suppose a valuer bids on a painting that she intends for her private use. Her valuation of the painting depends on her personal tastes, her intended use of the painting, and other considerations that are unique to her.[58] If she learned that other valuers placed a different value on the same painting, it might change her bidding strategy but would not change her valuation.[59]

SHAPIRO, COURTS: A COMPARATIVE AND POLITICAL ANALYSIS 186, 192 (1981) (stating that the prospect of winner-take-all outcome provides an "essential corrective for the tendency to overclaim").

56. *See, e.g.*, PAUL MILGROM, PUTTING AUCTION THEORY TO WORK 157–62 (2004); R. Preston McAfee & John McMillan, *Auctions and Bidding*, 25 J. ECON. LIT. 699, 705 (1987); *see also* Orley Ashenfelter & Kathryn Graddy, *Auctions and the Price of Art*, 41 J. ECON. LITERATURE 763 (2003); Peter Cramton & Alan Schwartz, *Using Auction Theory to Inform Takeover Regulation*, 7 J.L. ECON. & ORG. 27 (1991). *See generally* Eric Maskin, *The Unity of Auction Theory: Milgrom's Masterclass*, 42 J. ECON. LITERATURE 1102 (2004) (discussing the achievements and limitations of Milgrom's auction theory within the field of economics).

57. Represented mathematically, independent valuation describes a situation in which each valuer's valuation of an item is an independent draw from some probability distribution $F(v_i)$ where $v_i = v^1, v^2, v^3, \ldots v^n$. Thus, v_j represents the range of valuations held by valuers in the probability distribution. The holder of v^1 will not change her valuation if she learns that another valuer holds v^2. In such a market, there will be no change in the valuation of an item, even if all valuers revealed everything they knew (this, incidentally, is one definition of a perfectly "efficient" market. *See* Mark Rubinstein, *Securities Market Efficiency in an Arrow-Debreu Economy*, 65 AM. ECON. REV. 812, 823 (1975)). A market is thus "nonadjusting" when all valuers of the item hold independent valuations.

58. *See generally* Louis Kaplow & Steven Shavell, *Property Rules Versus Liability Rules: An Economic Analysis*, 109 HARV. L. REV. 713, 760 (1996) (discussing "idiosyncratic value").

59. *See generally* BENJAMIN GRAHAM, THE INTELLIGENT INVESTOR: A BOOK OF PRACTICAL COUNSEL 108–09 (4th rev. ed. 1985) (discussing the "parable" of "Mr. Market," in which market information does not affect the holder's valuation). In our database, the independent value for valuing principal was most often manifested as book value (a value assigned internally by the firm's accountants) or ownership testimony regarding a particular or idiosyncratic use of the property.

A valuer is using the common value when her valuation of an item is a best guess of how others will value the item in some common market.[60] Common value models assume that the item has "exactly the same value to each bidder,"[61] and that the variation in bids is explained by bidders' incomplete information regarding that value. Take, for example, an art dealer who bids for a painting she intends to resell. Her valuation is based on her information about what the painting will fetch in the resale market. Her valuation of the item will almost certainly change as she learns more about the valuations of others.

Using these categories of valuation, we assigned a code to each instance of valuation in the database. The possible codes ranged on a continuum from extreme common value to extreme independent value. Initial coding of the data resulted in assigning a number from "1" to "8" (with "1" representing extreme common value and "8" representing extreme independent value) for each valuation by courts, debtors, and creditors.

Subsequent statistical analysis[62] showed that the valuation standard could be collapsed into three categories: common (initial codes 1 and 2), mixed (initial codes 3, 4, and 5), and independent (initial codes 6, 7, and 8).

Independent values for interest rates most often took the form of pre-petition contract rates (a rate bargained for at some point in time historically distant from the time of the adjudicated valuation) or statutory rates (what a legislature decreed was acceptable compensation for the time value of money at some time historically distant from the time of the adjudicated valuation). As placeholders for value, both pre-petition contract rates and statutory rates assume that the found value should be something other than (or in addition to) what a willing buyer, in open competition with many other willing buyers, would pay a willing seller, in open competition with other sellers, for this item in these circumstances at this time.

60.   Represented mathematically, a "common" valuation is a draw from some probability distribution $H(v_i|V)$ where $v_i = v^1, v^2, v^3, \ldots v^n$. The only difference between probability distribution H (common valuation) and probability distribution F (independent valuation) is that the H quantity contains a term "V" representing the reference point by which all components of v_i adjust their valuations. On this theory of value, each component of v_i is an estimate of "V," the item's true value. These estimates stand to improve as each valuer learns the valuations of others. This true worth theory of value is the one employed by Bob Lawless and Steve Ferris. See Lawless & Ferris, supra note 9, at 11–12.

61.   MILGROM, supra note 56, at 162. In our database, common value for valuing principal was most often manifested as valuation models constructed by expert witnesses and which utilized elements such as liquidation models, comparables, and discounted cash flow. Common values for interest rates most often involved models utilizing elements of formal risk analysis.

62.   Specifically, a multinomial logistic analysis, a likelihood ratio test, and a Wald test all showed that the conditional distribution of the valuation standard could be collapsed from eight categories to three. See infra Appendix (directing the reader to website access of documents). The fact that the research project went fishing for eight categories and ended up with three is, itself, an interesting finding. This finding may verify, with statistical analysis of real-world bankruptcy data, the robustness of auction theory's three-category conceptualization of valuation.

Figure 6 shows the courts' valuation standard (CtVS) in the 180 observations of the database, as coded on an eight-part continuum ranging from "1" (extreme common value) to "8" (extreme independent value). We see that the valuations performed by bankruptcy courts in the database were skewed toward the common pole of the continuum, with most of the valuations coded at "3" (common, but not extremely common).[63]

FIGURE 6
COURT'S VALUATION STANDARD: FREQUENCY

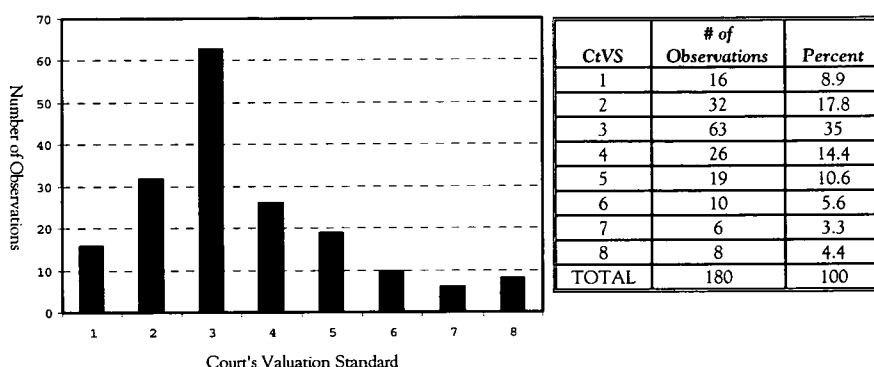| CtVS	# of Observations	Percent
1	16	8.9
2	32	17.8
3	63	35
4	26	14.4
5	19	10.6
6	10	5.6
7	6	3.3
8	8	4.4
TOTAL	180	100

Table A shows the distribution of the CtVS, broken out by whether the valuation contained a finance element—a calculation or discussion of the time value of money.

TABLE A
COURT VALUATION STANDARD BY FINANCE ELEMENT

	Frequency	Mean	Standard Deviation
Finance Element	74	3.20	1.48
No Finance Element	106	3.75	1.86

The presence of a finance element in the valuation tends to drive the CtVS closer to the common value.[64] As discussed in Part II.C.3, the observed cor-

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63.   These data tend to support a general impression that the cases in the database showed a rebuttable presumption in favor of more common-based valuation, a presumption which had to be overcome by the party seeking a more independent-based valuation.

64.   The t-test shows that $t = 2.0914$ with a probability of 0.0379 and that the observed difference is due to chance alone. So, the difference is statistically significant at the 5 percent level. *See also* the regression *infra* tbl.C.

relation between common valuation and the presence of a finance element is not only a feature of the sample (our database), but is also strong enough to allow an inference that the same correlation is true across the entire population.

Table B gives the mean and standard deviation of the CtVS for each of the federal circuits.

TABLE B
COURT VALUATION STANDARD BY CIRCUIT

Circuit	Mean	Standard Deviation	Frequency
1st	3.38	1.30	8
2d	3.96	2.01	27
3d	2.88	1.36	17
4th	3.40	1.96	15
5th	4.80	1.37	15
6th	3.17	1.63	24
7th	2.91	1.04	11
8th	3.32	1.95	19
9th	3.23	1.24	13
10th	3.50	2.43	6
11th	3.79	1.77	24
D.C.	3	n/a	1
*All Circuits	3.52	1.73	180

The Second, Fifth, and Eleventh Circuits have means that are statistically greater than "3."[65] There is some geographic distribution in the CtVS. Eliminating the circuits with fewer than ten observations (the First Circuit—Maine, Massachusetts, New Hampshire, Rhode Island, and Puerto Rico; the Tenth Circuit—Colorado, Kansas, New Mexico, Oklahoma, Utah, and Wyoming; and the D.C. Circuit), we can divide the data into a nonsouthern circuit group (the Second—Connecticut, New York, and Vermont; the Third—Delaware, New Jersey, Pennsylvania, and the Virgin Islands; the Sixth—Kentucky, Michigan, Ohio, and Tennessee; the Seventh—Illinois, Indiana, and Wisconsin; the Eighth—Arkansas, Iowa, Minnesota, Missouri, Nebraska,

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65.   A t-test confirms these results at the 5 percent level. This means that there is only a 5 percent chance that the observed correlation could have occurred by chance. The probability that the mean of the Second Circuit is not "3" is 0.0195; the Fifth Circuit's probability is 0.0002; and the Eleventh Circuit's is 0.0387. Descriptive statistics (frequency charts and tables reporting mean and standard deviation) for the CVS and DVS variables are maintained on the author's website. See infra Appendix.

North Dakota, and South Dakota; and the Ninth—Alaska, Arizona, California, Hawaii, Idaho, Montana, Nevada, Oregon, Washington, Guam, and Northern Mariana Islands) and a southern circuit group (the Fourth— Maryland, North Carolina, South Carolina, Virginia, and West Virginia; the Fifth—Louisiana, Mississippi, and Texas; and the Eleventh—Alabama, Florida, and Georgia).[66] The southern circuits score closer to "independent" value than the nonsouthern circuits, and that difference is statistically significant.[67]

2. Modeling Courts' Valuation

Our statistical model attempts to (1) explain how bankruptcy courts approach their task of valuing assets in cramdown proceedings, and (2) predict how bankruptcy courts will approach valuation in future cases. Accordingly, the dependent variable for these inferential statistics is the valuation standard of the bankruptcy court, represented as one of three categories: common, independent, or mixed. Taken together, the three categories of the dependent variable attempt to express a theory of value—an account of what a valuer is doing when it assigns value to an item.

We explain the court's valuation standard by reference to three independent variables: (1) debtors' valuation standard (DVS); (2) creditors' valuation standard (CVS)[68]; and (3) finance element (FINANCE), which is the variable for whether there is a calculation or discussion of the time value of money involved in the valuation.[69] We use an ordered logit model to estimate the relationship between the courts' valuation standard and the independent variables.[70]

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66. The nonsouthern/southern circuit division is constrained by the federal circuit groupings and thus only very imperfectly reproduces common conceptions about which states are southern and which states are nonsouthern. We count Kentucky and Tennessee in the "nonsouthern" group because those two states appear in the Sixth Circuit along with Michigan and Ohio; we count Arkansas and Missouri in the "nonsouthern" group because they appear with Iowa, Minnesota, Nebraska, North Dakota, and South Dakota in the Eighth Circuit.

67. The nonsouthern circuits have a mean of 3.324324 with a standard deviation of 1.679622 on the court's valuation standard variable. The southern circuits have a mean of 3.962963 and a standard deviation of 1.780004. The t-test shows that $t = 2.2472$ with a probability of 0.0260 that the observed difference is due to chance alone. The difference is statistically significant at the 5 percent level.

68. The three variables representing valuation standards (CtVS, DVS, and CVS) were originally coded on a continuum of "1" to "8." We subsequently converted that continuum to a three-part code for the court's valuation standard. We preserved the eight-part coding for the DVS and CVS because the larger number of distinctions among the explanatory variables made the estimation process more reliable and provided a better fit for the model.

69. The finance variable is coded as a binary unit: A code of "0" means there is no finance element in the valuation, and a code of "1" means there is a finance element in the valuation.

70. The model is presented and explained in the Appendix.

3. Findings

Table C presents results from the ordered logit model.

TABLE C
RESULTS: ORDERED LOGIT MODELS OF COURT VALUATION STANDARD

Variable	Estimates
DVS	0.933 (0.172)** [2.541]
CVS	0.853 (0.186)** [2.347]
FINANCE	1.613 (0.561)** [0.199]
Thresholds	
cut 1	3.552 (0.782)**
cut 2	9.890 (1.366)**
Log Likelihood	−54.724
BIC	−84.613
psuedo-r^2	0.475
Observations	119

Standard errors in parentheses
Odds ratios in brackets
** Significant at 1 percent level

Table C reports the effect that each independent variable has on the court's valuation standard, while holding constant the effect of all other independent variables. Table C lists three quantities for each independent variable: The top number is the coefficient for the natural log, or logit; the second number (in parentheses) is the standard error, showing the statistical significance of each result;[71] and the third number (in brackets) gives the odds ratio for each independent variable. Table C also reports estimates for the cutpoints.[72]

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71.   The stars represent statistical significance at the 1 percent level: The two threshold cutpoints and three independent variables (DVS, CVS, FINANCE) are all statistically significant such that the results are very unlikely to have been obtained by chance. Statistical significance at the 1 percent level means that the model captures the entire range of the data's distribution (assuming

The model shows that the explanatory variables DVS, CVS, and FINANCE predict CtVS variance at a statistically significant level. The pseudo-*r*-squared[73] is remarkably good for social science research, showing that the independent variables explain 47.5 percent of the dependent variable's observed variance. Overall, it appears that CVS, DVS, and FINANCE very usefully explain and predict the bankruptcy courts' valuation standard.

The odds ratios are quantities that help explain the model's results. An odds ratio[74] gives the odds that the dependent variable will be in one category versus another for every one-unit change in the independent variable.[75] For example, if CVS increases by one unit, the odds of CtVS being mixed becomes 2.347 times higher than the odds of CtVS being common. Likewise, when CVS increases by one unit, the odds of CtVS being independent becomes 2.347 times higher than the odds of CtVS being either common or mixed.

The results of the ordered logit models, as represented in Table C, support two interpretations. First, courts are much more likely to use a common valuation standard when the valuation contains a finance element (a calculation for the time value of money).[76] Why would the presence of a finance

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a bell-shaped distribution) except for the two extreme tails of 0.5 percent each. Adding "circuit" as an independent variable to the regression did not change the coefficients substantially.

72.   Ordered logit models assume proportional odds—the effect of the independent variable on the dependent variable is assumed to be proportional for all comparisons of categories. In other words, the slopes coefficients are identical across the estimated equations. In this case, two equations are estimated for each of the binary logistic regressions, yielding the coefficients denoted in Table C as "cut 1" and "cut 2." Though the slopes are the same, the probability curve is shifted to the left or right depending on the categorical comparison.

73.   The *r*-squared is a statistical measure of how well the model fits the data. In the case of our ordered logit model, a pseudo-*r*-squared is used. Roughly speaking, the *r*-squared tells us something about how much of the variation in the dependent variable is explained by the independent variables in the model.

74.   The odds ratios are the bracketed quantities in Table C. We get the odds ratio for a given independent variable by exponentiating its coefficient. For example, we exponentiate the FINANCE coefficient by taking base *e* (roughly 2.718) and raising it to the power of −1.613 (FINANCE's coefficient), yielding an odds ratio of 0.199.

75.   The odds of X happening is the probability of X happening divided by the probability of X not happening. An odds of "1" indicates that X stands a 50/50 chance of happening. An odds of greater than "1" indicates that the probability of X happening is greater than the probability of X not happening. And an odds of between "0" and "1" indicates that the probability of X not happening is greater than the probability of X happening.

76.   This finding can be discerned from Table C, which reports an odds ratio of 0.199 for the finance variable. The odds ratio of 0.199 means that, holding all other variables constant and looking just at the relationship between CtVS and FINANCE, if the observed valuation possessed a finance element, then (1) the odds of CtVS being mixed is 0.199 times the odds of CtVS being common; and (2) the odds of CtVS being independent is 0.199 times the odds of CtVS being either mixed or common. The data show that the presence of a finance element in the valuation pushes the courts' valuation standard sharply toward the common pole. This finding might be explained away as a meaningless coding error if cases containing a finance calculation had been automatically

element tend to drive valuations toward common standards? One possible expla-
nation is that judges deploy common valuation standards in high-complexity
cases,[77] but use more independent valuations in lower-complexity cases.

Second, the courts' valuation standard correlates strongly with the debt-
ors'[78] and creditors'[79] valuation standard. From this statistical result, we can
verify the unsurprising[80] proposition that lawyers frame the parameters of the
valuation dispute, and that judges typically work within those parameters.[81]
Since the data on parties' valuation standards is filtered through the courts'
interpretation, we cannot exclude an important alternative interpretation for
the correlation between courts' and parties' valuation standards: The court
may receive multiple valuation standards from the parties, but report only
those standards that it ultimately agreed with.[82]

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coded closer to common than to independent. The codes, however, demonstrate no such tendency.
Observations with finance elements show codes that span the range of valuation standards. *See infra*
Appendix (web access to spreadsheet of coding results, column BZ). Generally, a statistical model
cannot return sensible results unless there is meaningful variation in the data. Such variation would
be foreclosed by a coding error classifying finance element cases as common. Note also that many
cases that were coded closer to the independent pole contained a finance element. *See infra* note 78.

77.   *See generally* Elizabeth Warren, *Vanishing Trials: The Bankruptcy Experience*, 1 J. EMPIRICAL
LEGAL STUD. 913, 930–37 (2004) (contrasting high-complexity/low-volume bankruptcy cases
with low-complexity/high-volume cases). In our database, cases using common valuation standards
tended to involve competing and complicated models of the asset's valuation, constructed from the
ground up by dueling experts. Nothing in the database supports the proposition that common valua-
tion standards are used exclusively in business bankruptcy. Note, for example, the distribution
illustrated in Figure 6. Valuations in business cases, including valuations with a finance component,
can and do utilize standards closer to independent private value. The operative category for predict-
ing the occurrence of common versus private valuation standards seems to be high complexity versus
low complexity rather than business versus nonbusiness cases.

78.   Table C shows an odds ratio of 2.541 for DVS. This means that, holding all other
variables constant and looking just at the relationship between CtVS and DVS: (1) As DVS
increases by one unit, the odds of CtVS being mixed is 2.541 times higher than the odds of CtVS
being common; and (2) a one unit increase in DVS makes the odds of CtVS being independent
2.541 times higher than the odds of CtVS being either common or mixed.

79.   Table C shows an odds ratio of 2.347 for CVS. This means that, holding all other
variables constant, and looking just at the relationship between CtVS and CVS: (1) As CVS
increases by one unit, the odds of CtVS being mixed is 2.347 times higher than the odds of CtVS
being common; and (2) a one unit increase in CVS makes the odds of CtVS being independent
2.347 times higher than the odds of CtVS being either common or mixed.

80.   It should be noted that unsurprising is not *unimportant*. The empirical verification of an
anecdotally or experientially familiar proposition is part of what we should expect from applying the
scientific method to legal studies. *See supra* note 17 and accompanying text (explaining the
evidence-based law approach to doctrine).

81.   Of course, the observed correlation runs both ways: The values of DVS and CVS may
explain CtVS. But it is equally likely that debtors and creditors select their valuation approaches
based on their perception of courts' tendencies. Thanks to Bill Whitford for bringing this inter-
pretation to my attention.

82.   I thank Bill Whitford for this point.

D. Valuation Models

A valuation model is a particular argument or showing about the value of an item. The most common valuation models in the database were:

- Comparables, in which the modeler estimates an asset's value by looking to the known values of other, ostensibly similar assets.[83] Comparables are a common model for valuing residential real estate, in which a price of a house up for sale is compared to the recent sale prices of other houses of a similar location, size, and style.
- Discounted cash flow (DCF) models, which estimate the present value of future expected cash receipts and expenditures.[84] A DCF model is typically generated by a financial professional who is introduced as an expert witness. The financial expert will estimate the firm's future cash flows and then discount those expected future returns to present value through the use of a discount rate.
- Liquidation (also called cost, in some circumstances), in which the modeler estimates an asset's value by looking to what price the asset would fetch if sold.[85]
- Testimony by a current or prospective user of the asset regarding the valuation aspects of the intended use.
- A pre-petition contract rate agreed upon by the parties (used in the finance, or time value, aspect of the valuation).
- An accounting made for the risk profile of the party acquiring the asset (again used for the finance aspect of the valuation— higher risk yields an increased interest rate).
- General observations about market and economic conditions affecting the asset's value.

In addition to these categories, we also coded as unknown those observations in which the language in the opinion could not support the assigning of a code.

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83.   *See, e.g.*, MARK GRINBLATT & SHERIDAN TITMAN, FINANCIAL MARKETS AND CORPORATE STRATEGY 368 (1998).
84.   *See, e.g.*, RICHARD A. BREALEY & STEWART C. MYERS, PRINCIPLES OF CORPORATE FINANCE 73–77 (5th ed. 1996).
85.   *See, e.g.*, GRINBLATT & TITMAN, *supra* note 83, at 560.

Table D shows the valuation models offered by debtors and creditors, and accepted by the courts.

TABLE D

VALUATION MODEL OFFERED BY DEBTOR, CREDITOR;
ACCEPTED BY COURT

Valuation Model	Debtor	Creditor	Court
Comparables	24.4%	17.8%	23.3%
Discounted Cash Flow (DCF)	10.5%	8.9%	10.6%
Liquidation	4.4%	2.2%	3.9%
Testimony	6.7%	4.4%	3.9%
Pre-existing Contract Between Debtor and Creditor (Contract)	5.0%	2.8%	3.3%
General Market Conditions	0.6%	0.6%	1.7%
Comparables & Risk Profile	8.9%	10.6%	16.1%
Comparables & Liquidation	5.0%	0.6%	4.4%
DCF & Comparables	7.8%	8.9%	9.4%
DCF & Comparables & Liquidation	3.3%	1.7%	2.2%
General Market Conditions & Contract	0.6%	0.0%	0.6%
DCF & General Market Conditions	0.0%	0.0%	0.6%
DCF & Liquidation	0.6%	1.1%	1.1%
Comparables & General Market Conditions	0.6%	0.0%	1.7%
Comparables & Contract	1.1%	0.6%	1.1%
Comparables & Testimony	0.0%	0.0%	0.6%
Other	6.1%	3.9%	7.2%
DCF & Comparables & Other	0.0%	0.0%	0.6%
Comparables & Other	0.0%	1.1%	1.1%
Other & Testimony	0.0%	0.0%	0.6%
Unknown	14.4%	35.0%	5.6%
TOTAL	100%	100.0%	99.6%[86]

One way of organizing the data on court acceptance is to break the valuation models into two groups, roughly generalizing between hard valuation models (including DCF, Liquidation, Comparables, Risk Profile, and Pre-existing Contract) and soft valuation models (including Testimony and General Market Conditions).[87] Table E shows the distribution of court acceptance, broken out into hard and soft groups.

86. Column total does not equal 100 percent because of rounding.
87. Thanks to Neil Komesar for help on this point.

TABLE E
COURT ACCEPTANCE OF VALUATION MODELS:
DISTRIBUTION BY CIRCUIT

Circuit	Hard	Soft	Total Observations
1st	8 (100%)	0	8
2d	23 (95.83%)	1 (4.17%)	24
3d	13 (92.86%)	1 (7.14%)	14
4th	9 (100%)	0	9
5th	7 (87.50%)	1 (12.50%)	8
6th	20 (95.24%)	1 (4.76%)	21
7th	11 (100%)	0	11
8th	16 (94.12%)	1 (5.88%)	17
9th	10 (100%)	0	10
10th	4 (100%)	0	4
11th	13 (72.22%)	5 (27.78%)	18
D.C.	1 (100%)	0	1
TOTAL	135 (93.10%)	10 (6.90%)	145

The data show a preponderance of hard valuation models, as distributed across the federal circuits.[88] The data may offer some modest support for the proposition that courts in the southern circuits are more inclined to accept soft valuation models than are courts in the nonsouthern circuits.[89]

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88. For empirical work using the federal circuits as a unit of analysis, see generally Tracey E. George, *The Dynamics and Determinants of the Decision to Grant En Banc Review*, 74 WASH. L. REV. 213 (1999), and Tracey E. George & Michael E. Solimine, *Supreme Court Monitoring of the United States Courts of Appeals En Banc*, 9 SUP. CT. ECON. REV. 171 (2001).

89. For a list of the states or territories that comprise each circuit, see *supra* text accompanying notes 65–67.

E.   Self-Organization

These data are useful for testing the proposition that the U.S. bank-
ruptcy legal system might be a self-organizing system—a system whose con-
tent contains patterns not fully explained by exogenous dynamics.[90] This
subpart presents the evidence for self-organization as observed in three
categories of variables: (1) the use of valuation standards by courts, debtors,
and creditors; (2) the use of valuation models by debtors and creditors; and
(3) courts' acceptance of experts.

1.   Valuation Standards

If a system's content self-organizes, we should be able to measure a decline
over time in the variability of some of the system's variables. That is, we treat
variation itself as a structural variable to be measured and observed.[91]
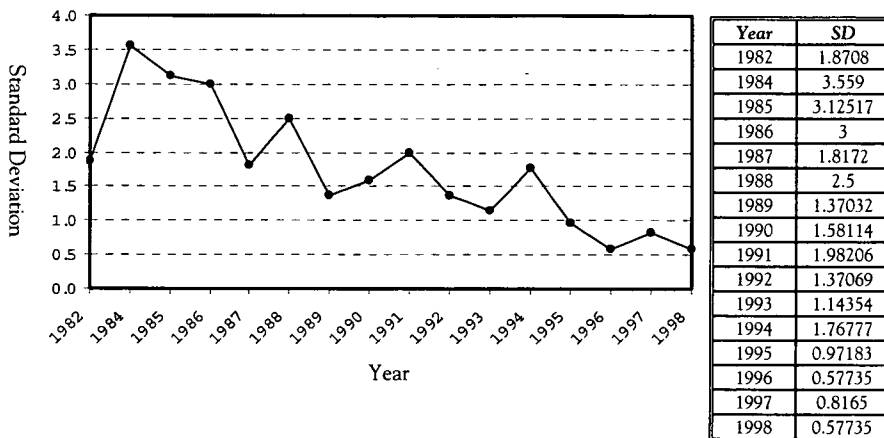
The measure of a variable's variation is its standard deviation. Just as
the mean, median, and mode say something about the central tendency of a
distribution, the standard deviation shows how that central tendency was
achieved—with relatively larger standard deviations signifying a wide range
of variation, and relatively smaller standard deviations signifying a smaller
range of variation. If the standard deviation of a variable declines over time,
this may indicate a sort of learning or patterning of system content over
time. Such patterns, unless explained by extra-system forces—such as control
by appellate courts or Congress—may constitute evidence of self-organization.

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90.   *See* Trujillo, *supra* note 1, at 519–43 (explaining the process of self-organization).
91.   The field of sociolinguistics has contributed much to my understanding of the construc-
tion of variation as a variable in order to study the change of a social artifice over time. *See generally*
ALESSANDRO DURANTI, LINGUISTIC ANTHROPOLOGY 79 (1997) ("Whereas the realization of such
variability convinced formal grammarians to ignore it by establishing an idealized homogene-
ity ... sociolinguists decided to *face variability* and make it the subject matter of their investigation.")
(emphasis added); J.K. CHAMBERS, SOCIOLINGUISTIC THEORY: LINGUISTIC VARIATION AND ITS
SOCIAL SIGNIFICANCE 11 (1995) ("[T]he admission of the variable as a structural unit in linguistic
analysis ... represents a breakthrough of considerable magnitude in linguistic theory ....").

Figure 7 shows the standard deviation of the CVS over time.[92]

FIGURE 7
CVS: DESCRIPTIVE

| Year	SD
1982	1.8708
1984	3.559
1985	3.12517
1986	3
1987	1.8172
1988	2.5
1989	1.37032
1990	1.58114
1991	1.98206
1992	1.37069
1993	1.14354
1994	1.76777
1995	0.97183
1996	0.57735
1997	0.8165
1998	0.57735

As a matter of descriptive statistics, it appears that the variability in the CVS declined markedly over time.

We can see whether it is proper to infer that the decline in the creditors' standard deviation, observed in the sample, is also a characteristic of the population. We do this by performing a standard linear regression.[93] The results of the regression are displayed in Figure 8.

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92. In Figures 7–12, the y-axis shows the standard deviation of the relevant valuation standard, as measured on the "1" (common) through "8" (independent) scale. The x-axis shows the year.

93. We can use a linear regression here, but we could not for the model of valuation standards presented in Table C. Here our dependent variable is the sample variance—a variable measured in cardinal numbers. It was the ordinal measure of the dependent variable in Table C that required the use of an ordered model. *See infra* Appendix. The formula for this standard regression is $y = A + Bx$, where y is the dependent variable (the variance of CVS), A is the y-intercept of the line, B is the slope of the line, and x is the independent variable (the year). This model estimates a line describing the relationship between the dependent and independent variables in the population. A negative slope provides evidence of a negative relationship between variation and time.
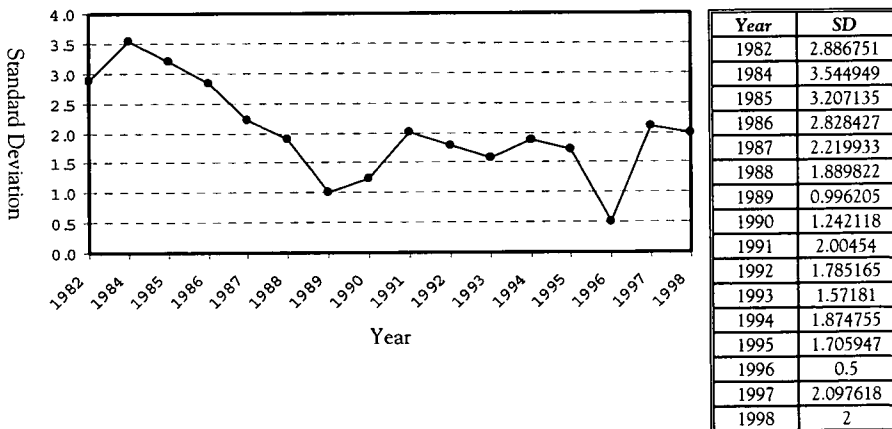
FIGURE 8
CVS: INFERENTIAL
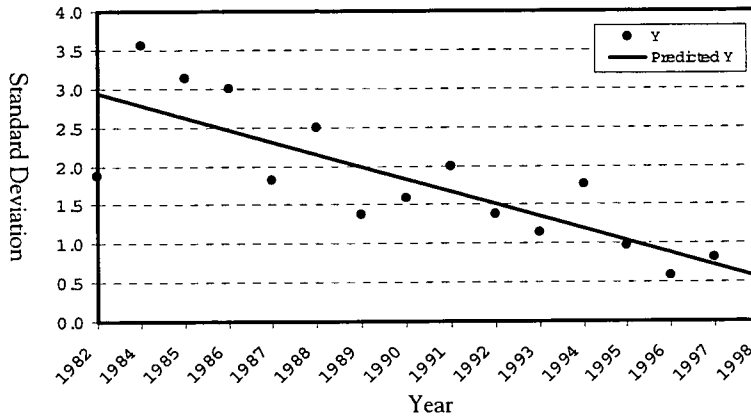
The line estimated by the linear regression model explains the behavior of the sample data quite well,[94] allowing us to conclude that the CVS variable does organize over time.

Figure 9 shows the standard deviation of the DVS over time.

FIGURE 9
DVS: DESCRIPTIVE

| Year	SD
1982	2.886751
1984	3.544949
1985	3.207135
1986	2.828427
1987	2.219933
1988	1.889822
1989	0.996205
1990	1.242118
1991	2.00454
1992	1.785165
1993	1.57181
1994	1.874755
1995	1.705947
1996	0.5
1997	2.097618
1998	2

94. $r^2 = 0.6719$, which is quite strong given the limitations of social science data.

This figure, while not as clear as Figure 7, indicates some decline in the sample variance over time. We can test the statistical significance of that decline by applying the same standard linear regression as above. Regression results are shown in Figure 10.

FIGURE 10
DVS: INFERENTIAL
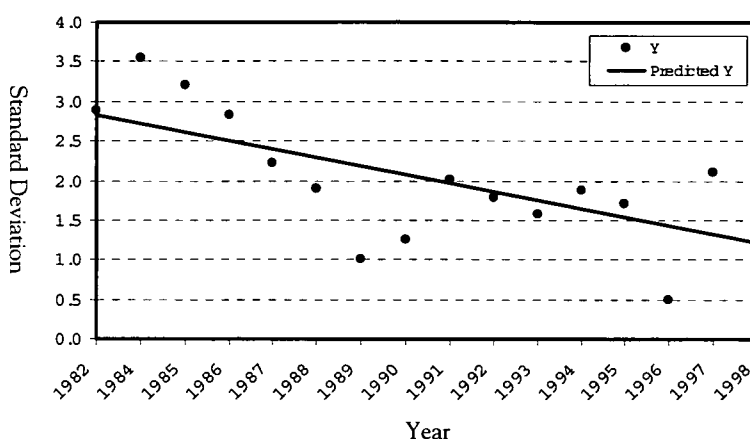
Year

The line estimated by the linear regression model explains the behavior of the sample data,[95] allowing us to conclude that the DVS variable organizes over time.

Note that we do not see the parties' valuation standard converge on a single numeric value, or mean, within a given year. Such a finding would have indicated that the parties gradually came to agree on a particular valuation theory, something quite unlikely to happen given that the facts of every case and the litigation goals of every party are different. What we do see, however, is a year-to-year shrinkage of the range of variation within the debtors' network and also within the creditors' network.[96] It would seem as

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95. $r^2 = 0.4097$.

96.   System-wide declines in standard deviation can be evidence of system-wide convergence on a given form. Much more likely, however, such declines in standard deviation would signal only a convergence within districts, while sharp differences between districts would remain. Suppose a system with two districts (say, the Western District of Wisconsin and the Eastern District of Wisconsin). In 1979, there was substantial variation in both districts about how to value a car (represented mathematically as eleven values: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0). By 2005, the Western District had settled on "look it up in the bluebook," while the Eastern District settled on "get it appraised by an expert" (represented as two clusters totaling eleven values: six values of 0.2 and five values of 0.6). The standard deviation for that system will have declined

if there is increasingly perfect communication among the actors comprising
the debtors' network, and among the actors within the creditors' network,
regarding the range of legitimate valuation approaches to be utilized in a given
market environment.[97] Put another way, the nature of the organization over
time that we observe in the parties' valuation standard demonstrates not
"uniformity"—parties agreeing on one valuation theory—but rather "unity"—
parties within a given practice network becoming diachronically more coher-
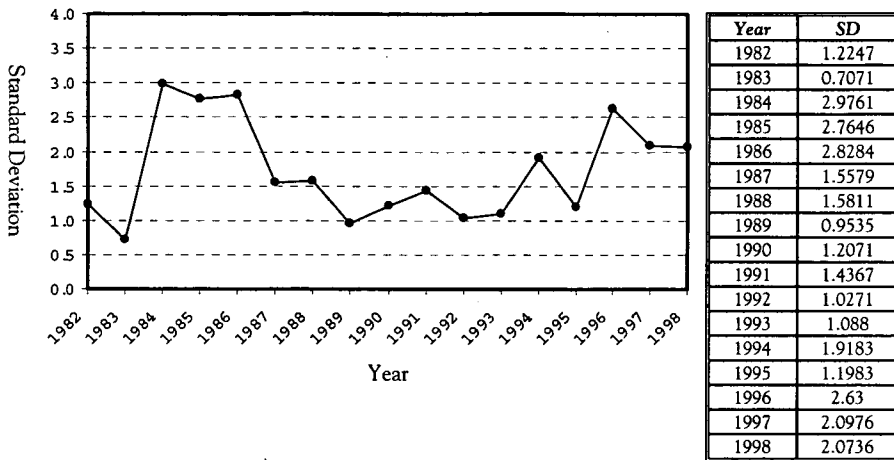ent in the range of approaches they attempt.[98]

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from 1979 ($SD = 0.3317$) to 2005 ($SD = 0.2089$), even as the interdistrict variation will have
sharpened. Other research suggests that the pattern of intradistrict convergence/interdistrict varia-
tion accurately describes the bankruptcy legal system. See Trujillo, supra note 1, at 510–15, and
sources cited there. Sharp and persistent interdistrict variations, far from being incompatible with
self-organizing dynamics, are a common product of such dynamics. Moreover, because the present
project is devoted to understanding bankruptcy system dynamics, it is beyond the scope of the
present work to opine about whether interdistrict variation is a good or bad thing. A separate set of
criteria is necessary to judge the goodness or badness of given variations, and certainly some of the
variations would be normatively better than others. As an analogy, consider a meteorologist engaged
in basic research about weather system dynamics. This basic research tries to explain why storms
occur when and where they do. The research has nothing to say about whether one storm is bad
because it caused a destructive flood, while another storm is good because it brought needed moisture
to a parched farmland. Understanding basic system dynamics and normatively evaluating specific
system events are simply two different jobs.
    97.   Within the valuation standard variable, we do not see a convergence of the valuers on a
given form, such as common or independent, such that a certain valuation standard becomes the
norm. Rather, we see a shrinkage of variation, itself understood as a variable. The low number of
observations in this database precludes the strategy of analyzing trends at the district level. Approaches
to the low-n problem for studying legal system dynamics are discussed infra notes 111–112 and
accompanying text.
    98.   See HERBERT KAUFMAN, THE FOREST RANGER: A STUDY IN ADMINISTRATIVE
BEHAVIOR 203 (1960) (finding "unity" without "uniformity" in an organizational analysis of the
U.S. Forest Service).

While the valuation standards for the debtors and the creditors seem to show some patterning over time, the courts' valuation standard shows no such organization. Figure 11 shows the standard deviation of the CtVS over time.

FIGURE 11
CTVS: DESCRIPTIVE

Year	SD
1982	1.2247
1983	0.7071
1984	2.9761
1985	2.7646
1986	2.8284
1987	1.5579
1988	1.5811
1989	0.9535
1990	1.2071
1991	1.4367
1992	1.0271
1993	1.088
1994	1.9183
1995	1.1983
1996	2.63
1997	2.0976
1998	2.0736

The eye detects no systematic decline in the sample variance, and this conclusion is borne out by the regression shown in Figure 12.

FIGURE 12
CTVS: INFERENTIAL
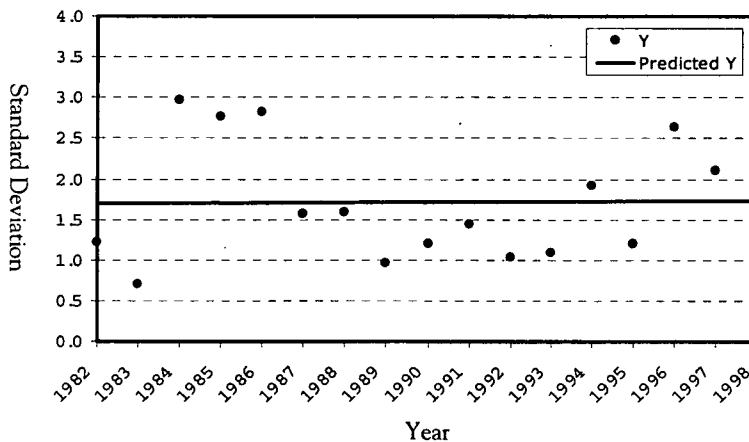
We can safely conclude from Figure 12 that the CtVS shows no organization or patterning over time.[99] Thus, while we observe some ordering phenomenon in the valuation standard variable among debtors and creditors, we observe no such phenomenon among the courts.[100]

2.   Parties' Valuation Models

In a self-organizing system, we would also expect to see forms introduced from outside the system, as well as a process of competition by which some of these forms achieve normative status.[101] We perform time-series analyses of the parties' use of particular valuation models to test for the presence of such form innovation and norm emergence.
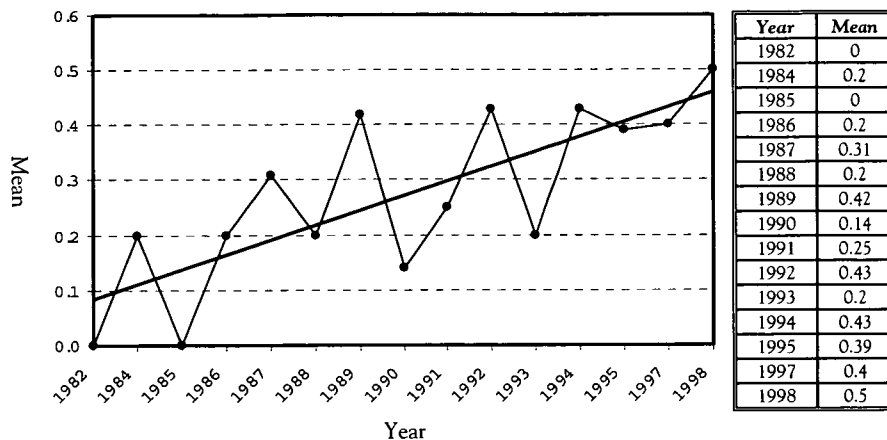
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99.   The slope of the regression line is not negative, and the r^2 is 0.0003. One simple conclusion we can draw from this absence of a decline in courts' variance is that any ordering that we observe in this area of valuation is likely not exogenous (that is, in response to congressional or appellate court directives). If there were exogenous ordering, we would likely see a decline in court variance.

100.   Figures 7–12 organize party-based variables and court-based variables along the matrix of time. When time is the explanatory variable, we see that party-based variables and court-based variables perform differently. This is an intriguing finding, perhaps disclosing an important mathematical/structural feature of legal system dynamics. Much further analysis (probably using a model generating a large number of observations over a long period of time, thus overcoming the profound limitations of the instant database) will be needed before reaching any reliable conclusions. As a tentative first pass, consider the problem from two related perspectives: First, why would parties and courts behave in different ways? Using the language of dynamical systems we can say that we have observed an attractor, or pattern, among the CVS and DVS variables. This attractor is explained by the parameters of the system, which parameters must themselves be off the attractor. That is, the parameters determine the shape of the attractor, but are not contained within the plot of the attractor. Here the attractor, the pattern we observe, is reported litigant behavior. Courts, and more specifically, court allocation of the scarce resource of official recognition of party-introduced forms, are part of the system parameters that determine the shape of the attractor. Second, why would we see parties' variance shrink over time, but not courts' variance? Suppose a dynamics in which two opposing networks of parties (Network A and Network B) work independently to develop forms that will compete for the scarce resource of official recognition allocated by courts. Within each of the two networks, we would anticipate observing the emergence of norms, with the norms of Network A differing from the norms of Network B. The two networks then pose their norms before a court, which picks between the two. This process, over many iterations, would likely produce patterns similar to the patterns produced by self-organizing critical systems—we would see clustering of system content (a decline in party variance), with no long-term predictability of the size, shape, or location of those clusters (no decline in court variance). Conversations with Bill Clune and Clint Sprott have contributed much to the early development of these points.

101.   See Sprott, supra note 1, at 329; J.C. Sprott et al., Coexistence and Chaos in Complex Ecologies, 335 PHYSICS LETTERS A 207–212 (2005), available at http://sprott.physics.wisc.edu/pubs/paper293.pdf.

Figure 13 shows the debtors' use of DCF models during the years of the sample.[102]

FIGURE 13
DEBTORS' USE OF DCF VALUATION MODELS

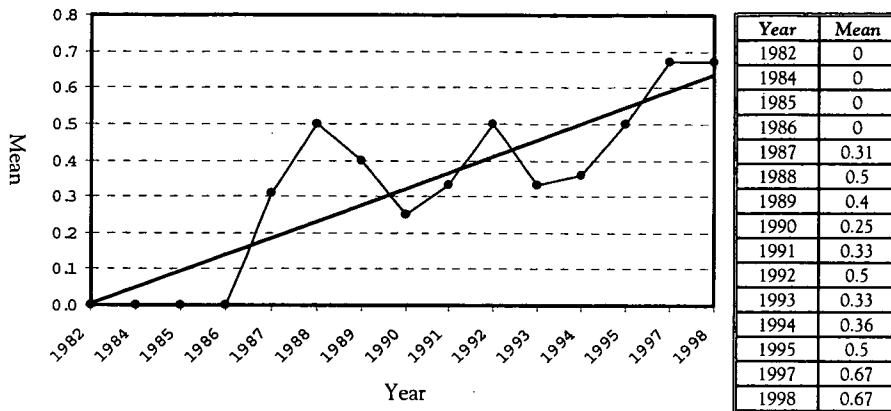| Year	Mean
1982	0
1984	0.2
1985	0
1986	0.2
1987	0.31
1988	0.2
1989	0.42
1990	0.14
1991	0.25
1992	0.43
1993	0.2
1994	0.43
1995	0.39
1997	0.4
1998	0.5

This figure shows simple descriptive evidence that debtors increased their use of DCF models during the years of this sample. A mean of "0" indicates that debtors did not use DCFs at all in the given year, while a mean of "1" would indicate that debtors used DCFs in every instance of a given year. The data tell a story of the DCF steadily gaining usage among debtors' networks, until the point where it becomes a regular "arrow in the quiver" of debtors' valuation arguments.

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102.   In Figures 13–16, the variable is a binary and so only the mean is presented. Where the variable is a binary, the standard deviation is simply a transformation of the mean, and so the standard deviation adds no information to the mean's description of the distribution.

Figure 14 charts the creditors' use of the DCF model over the same time period, showing an even stronger upwards trend than the debtors' data.
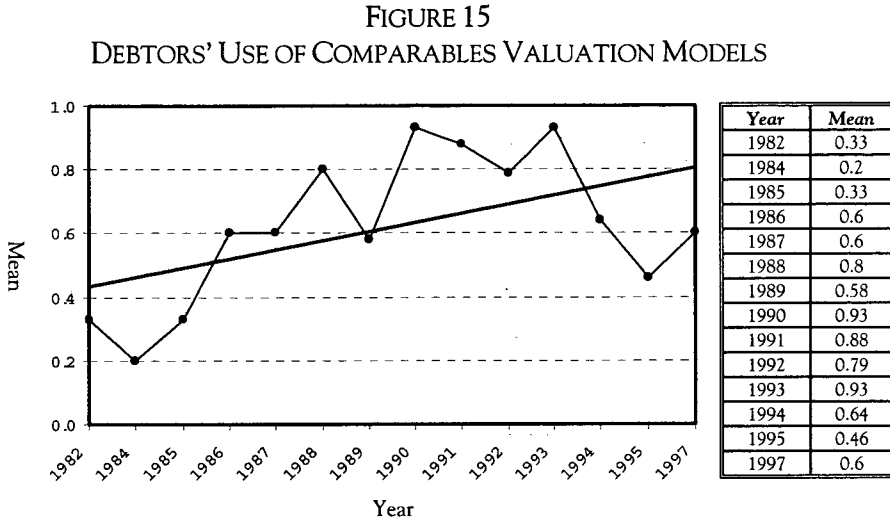
FIGURE 14
CREDITORS' USE OF DCF VALUATION MODELS

Year	Mean
1982	0
1984	0
1985	0
1986	0
1987	0.31
1988	0.5
1989	0.4
1990	0.25
1991	0.33
1992	0.5
1993	0.33
1994	0.36
1995	0.5
1997	0.67
1998	0.67

The data support an interpretation that the DCF valuation model entered the legal system in the late 1980s,[103] and gradually rose to the level where it became a common way of arguing about value.
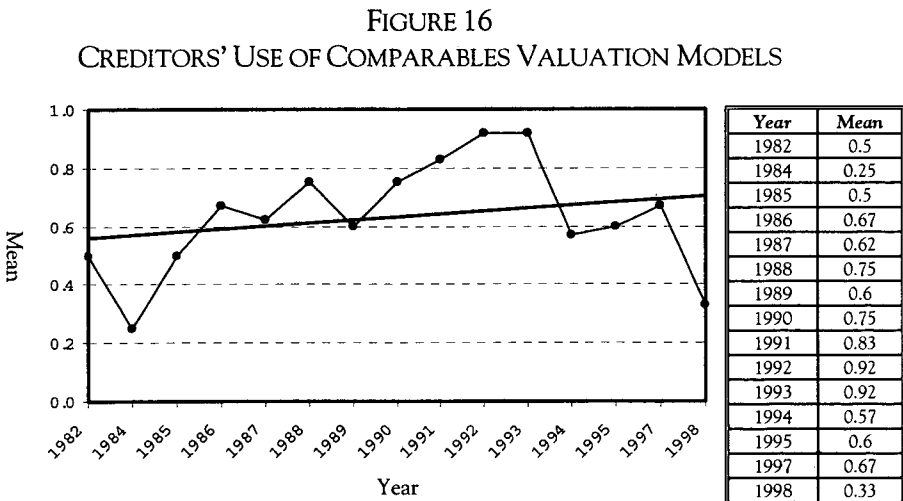
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103.   By the late 1980s, DCF models were widely used in financial networks. I have previously argued that the DCF presents an example of intersystem form transportation, in which a form that is well established in one network (a financial-based network) is received into another network (a legal network). See Trujillo, supra note 1, at 555, 558–59.

Figure 15 shows the debtors' use of comparables valuation models over time.

FIGURE 15
DEBTORS' USE OF COMPARABLES VALUATION MODELS

Year	Mean
1982	0.33
1984	0.2
1985	0.33
1986	0.6
1987	0.6
1988	0.8
1989	0.58
1990	0.93
1991	0.88
1992	0.79
1993	0.93
1994	0.64
1995	0.46
1997	0.6

Note the period from 1990–1993, during which the mean is noticeably higher than the remainder of the sample. During this period, we can conclude that debtors' use of comparables had become normative.

Figure 16 shows the data for creditors' use of comparables.

FIGURE 16
CREDITORS' USE OF COMPARABLES VALUATION MODELS

Year	Mean
1982	0.5
1984	0.25
1985	0.5
1986	0.67
1987	0.62
1988	0.75
1989	0.6
1990	0.75
1991	0.83
1992	0.92
1993	0.92
1994	0.57
1995	0.6
1997	0.67
1998	0.33

Creditors' data show a trendline that is essentially flat, indicating that comparables were part of the creditors' arsenal throughout the sample period. Noting the same period from 1990–1993 that we saw in the debtors' data, we can conclude that parties' use of comparables rose to a normative level before tapering off somewhat during the mid-1990s.

Generally speaking, we can say that the introduction of such forms as the DCF and comparables support a subjective impression of the database that, following some initial unfamiliarity, bankruptcy actors became more sophisticated in their handling of valuation disputes over time.[104]

Such a process of adaptation or learning is a form of self-organization. Adaptation, including the emergence of new systemic features, occurs as system actors acquire expertise by importing valuation forms from outside the legal system, and gradually coming to master those forms.[105] From a courts-based perspective, one could say that the judges' exercise of their discretion gradually became more transparent. From a clients-based perspective, one could say that the risk—defined as the degree of dispersion of possible outcomes[106]— of submitting a dispute to adjudication declined over time.

Note that an increase in the valuation sophistication of bankruptcy actors does not mean that those actors move closer to a common valuation standard. The phenomenon of rising sophistication in conducting valuation determinations—for example, the importation of technical financial models into the courts[107]—seems largely independent of whether the valuer uses a common or independent theory of value.[108]

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104.   See, e.g., Butler, supra note 9, at 342 (noting an increase in bankruptcy valuation proceedings as "bankruptcy practitioners become more practiced in their art"); Fortgang & Mayer, supra note 9, at 1061 (sketching basic finance principles for bankruptcy valuation proceedings). The articles from Butler (a practitioner in a South Carolina law firm) and Fortgang and Mayer (practitioners at Wachtell, Lipton, Rosen & Katz in New York City) are themselves data indicating the bar's gradually increasing financial sophistication in trying valuation disputes.

105.   This learning occurs first as judges become educated by experts about financial valuation methods, and then, as similar issues recur multiple times, deepening the courts' experience and increasing their confidence to criticize and depart from experts' models. See ELIOT FREIDSON, PROFESSIONAL POWERS: A STUDY OF THE INSTITUTIONALIZATION OF FORMAL KNOWLEDGE, at xi (1986) ("[W]hile the institutionalization of knowledge is a prerequisite for the possibility of its connection to power, institutionalization itself requires the transformation of knowledge by those who employ it."); Trujillo, supra note 1, at 536–39, 547–53 (discussing intersystem "transportation-cum-distortion" of forms).

106.   See, e.g., WILBUR G. LEWELLEN, THE COST OF CAPITAL 17 (1969).

107.   See, e.g., Oliver E. Williamson, Chester Barnard and the Incipient Science of Organization, in ORGANIZATION THEORY: FROM CHESTER BARNARD TO THE PRESENT AND BEYOND 172 (Oliver E. Williamson ed., expanded ed. 1995) (discussing "managerial norms").

108.   For example, the DCF is a sophisticated valuation technique, yet is based entirely on projections of an individual firm's peculiar use of property.

3. Summary of Findings

The data offer preliminary support for the conclusion that some aspects of the U.S. bankruptcy legal system show a tendency to self-organize. Conclusive evidence of self-organizing dynamics in a legal system could have substantial jurisprudential significance.

We know that simple deterministic dynamics do not explain the data we observe in legal systems. Since the decline of legal formalism, the dominant mode of explanation has been to attribute a randomness, or nondeterminism, to legal system dynamics and to suggest that any observable patterns are due to exogenous ordering—such as decisionmaker bias—that affect legal ordering intersystemically.[109] Evidence of self-organizing dynamics suggests the possibility that at least some of the patterns we observe are generated by deterministic dynamics operating intrasystemically.

Legal realism and critical legal studies moved legal explanation from simple determinism to nondeterminism plus exogenous ordering. Now, evidence of self-organization may move legal explanation back in the direction of a complex determinism. Self-organization suggests that at least some patterns in legal data are generated by a complex and nonlinear deterministic dynamics—in other words, chaos.[110] Put another way, the data in this study may help to falsify quantitatively the jurisprudential hypothesis that legal dynamics are random rather than chaotic.

Any such argument based on these data is, however, far from conclusive because of the low number of observations—180, with some variables possessing fewer than 180 observations. These data simply are not sufficient to take advantage of all that time-series analysis has to offer in substantiating robust conclusions about the dynamics of the legal system.[111]

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109. The Efficient Capital Markets Hypothesis (ECMH), as a form of financial explanation, follows a similar course. The ECMH attributes a randomness to the trade-to-trade movements of stock prices and argues that only exogenous forces—the surprise of new information—explain price movements. *See, e.g.*, Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, 25 J. FIN. 383 (1970).

110. *See* SPROTT, *supra* note 1, at 20. Sprott notes that:
    Chaotic systems . . . have a number of characteristics:
      1.   They are *aperiodic* (they never repeat).
      2.   They exhibit *sensitive dependence on initial conditions* (and hence they are unpredictable in the long term).
      3.   They are governed by one or more *control parameters*, a small change in which can cause the chaos to appear or disappear [and]
      4.   Their governing equations are *nonlinear*.
Id.

111. *See id.* at 236–38 (discussing time-delayed space embedding).

This research on legal systems' self-organization can take at least three possible future directions. First, we can go about collecting much more data, in order to get the number of observations up to the point that will support robust conclusions. Second, we can pursue a number of methodologies that have been developed to assist in the analysis of low-observation datasets.[112] Finally, we can construct a mathematical model of the system dynamics we want to explain, and that model will generate sufficient observations to support a full time-series analysis.

CONCLUSION

This Article has presented some initial results from the bankruptcy cramdown valuation database. It also has set out to explain both the doctrine of bankruptcy valuation, and what that doctrine might indicate about the dynamics of the bankruptcy legal system.

Major findings of this study include the following:

1. Courts tend to split the difference in valuations much less frequently than we had expected (Figures 1–2).
2. Parties' valuations, initially close together, have moved further apart over time (Figures 3–5).
3. Bankruptcy courts are much more likely to use a common valuation standard when the valuation includes a calculation for the time value of money (Table C).
4. There seems to be some geographic distribution of courts' acceptance of valuation models, with courts in southern circuits more likely to accept soft valuation models, such as owner testimony or evidence of general market conditions, and nonsouthern circuit courts more likely to accept hard valuation models, such as discounted cash flow models, comparables, or liquidation (Table E).
5. There is evidence that bankruptcy system content tends to self-organize. Specifically, variation decreases over time (Figures 7–12), and we see evidence of form transportation and norm emergence (Figures 13–16).

Areas of future research based on these data may include an exploration of the relationship of valuation models to success, to find out whether certain arguments about valuation achieve client victory more often than others.

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112.   *See, e.g.,* John C. Gallant et al., *Estimating Fractal Dimension of Profiles: A Comparison of Methods,* 26 MATHEMATICAL GEOLOGY 455 (1994); Alberto Malinverno, *A Simple Method to Estimate the Fractal Dimension of a Self-Affine Series,* 17 GEOPHYSICAL RES. LETTERS 1953 (1990).

Another area of future research may involve the subset of observations classified as finance cases. These observations are rich in detail, and perhaps time-series analysis could yield important information about how the bankruptcy legal system has received and utilized quantification practices from outside the legal system.

More generally, these data, coupled with a methodology based on quantification and statistical inquiry, may put us in a position to make progress on a fundamental question in legal studies: whether legal system dynamics are nondeterministic.

APPENDIX: METHODOLOGY

A. Reliability

Most of the variables coded in this study are objective, such as date of case, presence of finance element, or amount of parties' and courts' valuation. The valuation standard variable, however, involves a subjective judgment made by the coder—deciding, for example, to code one observation as a "4" and another observation as a "3." Standard social science methodology directs that, when there is a subjective element involved in the coding, the research take special steps to test the reliability of the coding.

Accordingly, after the completion of the initial codes on which the statistics were based, a subset of the data for the dependent variable was independently recoded by different coders. The results of that reliability recode are presented in Table 1-A.

TABLE 1-A
RELIABILITY RECODE STATISTICS

Difference of +/-1

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-1	6
Percentage Difference	4.00%
Percentage Same	96.00%

Difference of +/-2

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-2	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/-3

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-3	2
Percentage Difference	1.33%
Percentage Same	98.67%

Difference of +/-4

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-6	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/-5

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-5	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/-6

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-6	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/-7

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-7	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference of +/-8

Total Number of Cases Recoded	150
Total Number of Cases with a Difference of +/-8	0
Percentage Difference	0.00%
Percentage Same	100.00%

Difference When Code Assigned No Value and Recode Assigned Value

Total Number of Cases Recoded	150
Total Number of Cases with a Difference	7
Percentage Difference	4.67%
Percentage Same	95.33%

Total Difference

Total Number of Cases Recoded	150
Total Number of Cases with a Difference	15
Percentage Difference	10.00%
Percentage Same	90.00%

The original coding process yielded 180 observations. The recode took fifty observations (27.8 percent of the observations in the database) and independently assigned codes for the three valuation standard variables—CtVS, DVS, and CVS. This process thus yielded 150 recodes.

Table 1-A shows that, of the 150 recodes, 135 received the same code as in the initial coding process, resulting in a recode reliability rate of 0.9, which is well within the boundaries of reliability for social science research.[113]

B.   Ordered Logit Model

Table C reports the results of an ordered logit model. This section explains the model and why such a model was needed.

How one measures the dependent variable determines the selection of the appropriate statistical model. A very common statistical model is the least-squares regression. Least-squares regressions, however, require that the dependent variable be presented in interval measures—cardinal numbers that specify the distance between each value. For example, the dependent variable height is expressed in interval measures because differences between values (say between 5'8" and 5'10") are fixed and meaningful.

The present study tries to explain how bankruptcy courts approach valuation. This dependent variable cannot be expressed with interval measures. We measure standards of valuation based on a set of conceptual categories. The values of our dependent variable are thus expressed not cardinally (with numbers such as 1 and 2, or 5'8" and 5'10"), but rather ordinally (with numbers such as 1st and 2nd). While the values of cardinal numbers are known absolutely, the values of ordinal numbers are known only by the relationships among the values, for example, the second largest amount, so that the distances between ordinal categories are not quantified.

Ordinal categories present a special problem for statistical model building. If data are presented in interval measures, the difference between one category and another is quantified by definition. But the statistician must take special steps to quantify the difference between ordinal categories.

Because the data in this study are ordinally measured, we need a statistical model equipped to estimate the intercepts—where one category ends and the next begins—for each category. This study uses an ordered logit model, which estimates the intercepts for each category of the dependent variable.

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113.   *See* PAUL E. SPECTOR, SUMMATED RATING SCALE CONSTRUCTION: AN INTRODUCTION 65–67 (Sage University Paper Series: Quantitative Applications in the Social Sciences No. 07-082, 1992).

For an ordinal variable with J categories, let the odds that the dependent variable (the y term in the equation below) will have a value less than or equal to a given intercept m, versus the odds that the dependent variable will have a value greater than a given intercept m, be:

$$\Omega_{\leq m_k | > m_k}(x) = Pr(y \leq m_k | x) / Pr(y > m_k | x)$$

where x is a vector of explanatory variables, and we estimate for $J-1$ thresholds. This equation merely defines the odds of an event—that the dependent variable will have a certain value relative to the independent variable—as a ratio of the probabilities for that event.

The log of the odds as defined in the first equation is thus equal to:

$$ln \ \Omega_{\leq m_k | > m_k}(x) = \tau_{m_k} + x\beta$$

where τ_{m_k} states the estimated thresholds, y estimates the intercepts along the y-axes, and β (as a vector of the coefficients) states the slope of the dependent variable as it moves along the x-axis. This equation predicts the value of the dependent variable given certain independent variables. Figure 1-1 graphically represents the parameters and predictions of the model.

FIGURE 1-1
ORDERED LOGIT MODEL OF CTVS: GRAPH AND EQUATION

x – values of independent variables
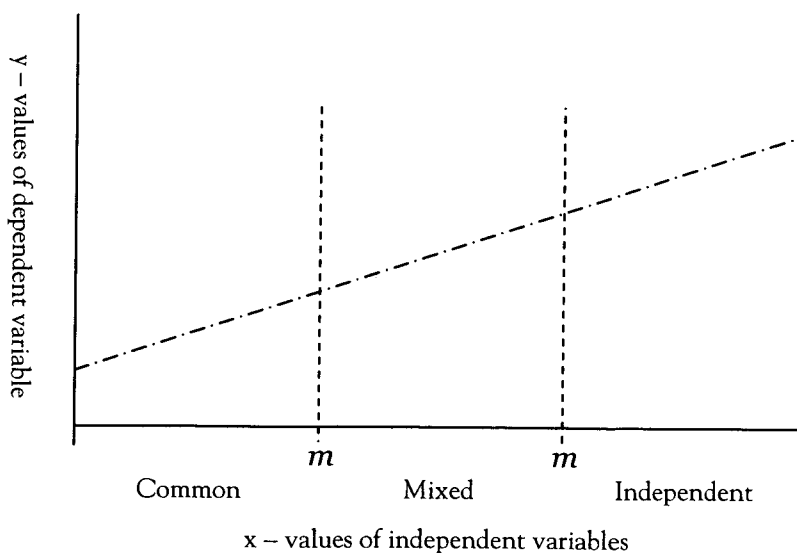
The equation $ln\ \Omega_{\leq m_k | > m_k}(x) = \tau_{m_k} + x\beta$ predicts the changing value of y (the courts' valuation standard) by:

1. Estimating the m_1 and m_2 cutpoints along the horizontal axis (modeled by the τ_{m_k} term);

2. Estimating the points at which the dependent variable intercepts the vertical axes (modeled by the y term); and

3. Estimating the slope of the dependent variable as it moves along the horizontal axis (modeled by the β term).

This is an ordered logit model. The natural logarithm of the odds—known as the logit—is a linear function of the independent variables. In order to account for the ordinality of the dependent variable, $J-1$ equations are estimated and the coefficients of the independent variables are constrained to be equal across equations.

C.   Access to Documents of Potential Interest

Documents useful for the verification and replication of this study are maintained at http://www.law.wisc.edu/facstaff/pubs.php?ID=429.

These documents include: (1) a list of the 388 cases that constituted the initial database; (2) the coding instrument; (3) instructions to coders; (4) the spreadsheet of coding results; and (5) an explanation of the statistical tests (multinomial regression, Wald, and likelihood ratio) that explain the reconfiguration of the valuation standard variable from an eight-point continuum to a three-point continuum.